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RIVERS – CONNECTING
MOUNTAINS AND COASTS

***23rd Arthur Thomas Ippen Lecture***

# EXPLORING NATURAL AND ANTHROPOGENIC IMPACTS ON FRESHWATER LENS ON SMALL OCEANIC ISLANDS

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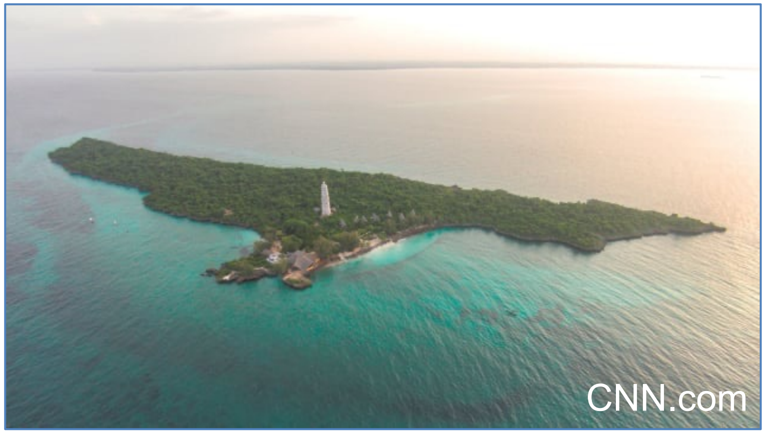
# Background

- Island: a body of land surrounded by water
- 200,000 islands (>0.1km2) on the Earth (~6.7% of the Earth's land area)
- Over two thirds of the world's countries (~130 countries) include islands
- More than 650 million inhabitants (~8% world's population)
- Continental island, oceanic island (volcanic island and coral island), and alluvial island in the ocean

# Natural Island


smithsonianmag.com

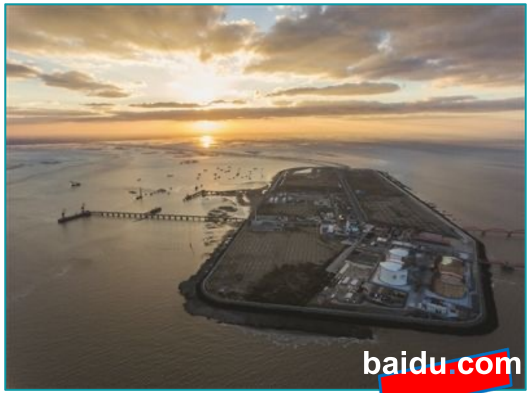

smithsonianmag.com


CNN.com


CNN.com

# Artificial Island

**Freshwater ?**



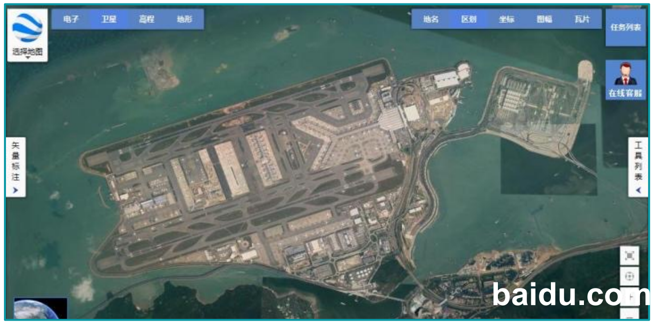
Sunshine island (CH
3 km$^2$

Treasure island (USA)
2 km$^2$

Port Island (JPN)
4.3 km$^2$

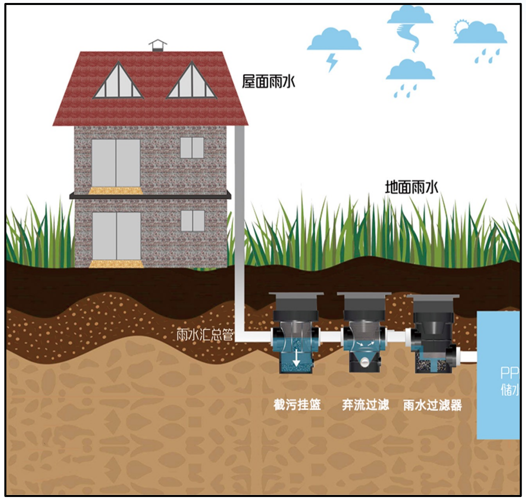
Palm Jumeirah (UAE)
12 km$^2$

Jurong Island (SGP)
32 km$^2$



Hong Kong International Airport (CHN)
12.6 km$^2$

# Freshwater Supply



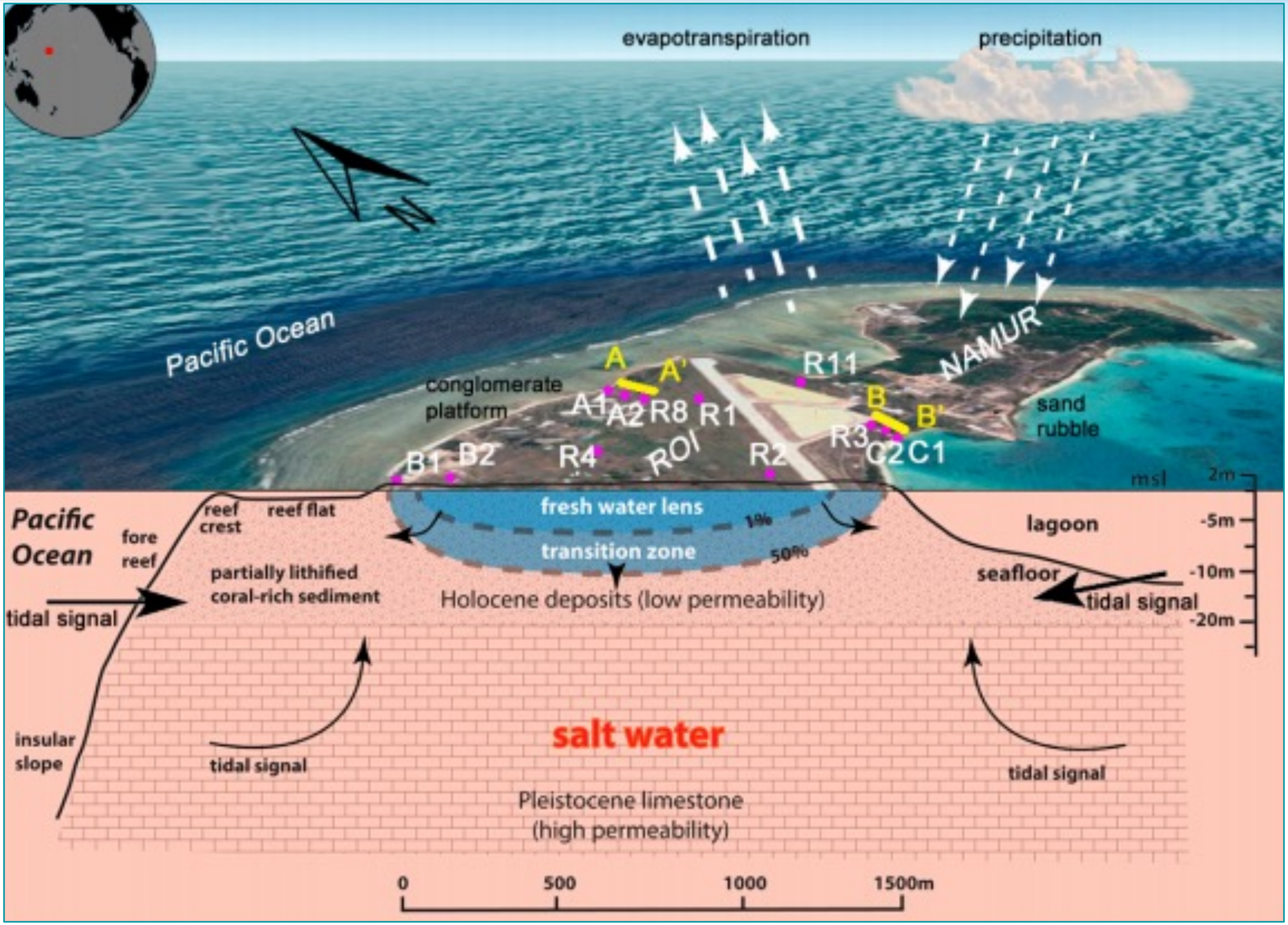
Rainwater Harvest

Desalination Plant

Boat Transport

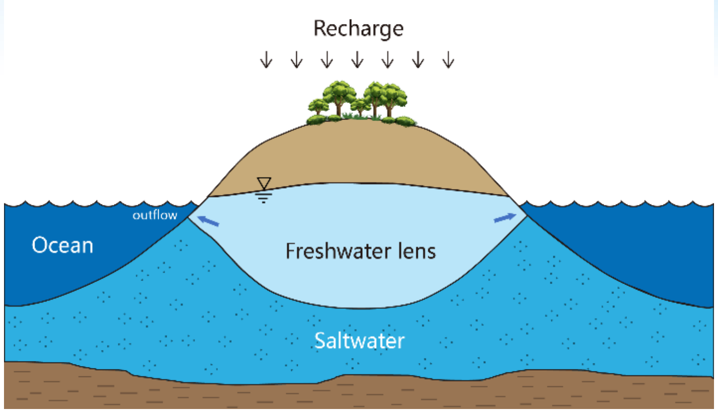
Reclaimed Water Use

# Freshwater Lens



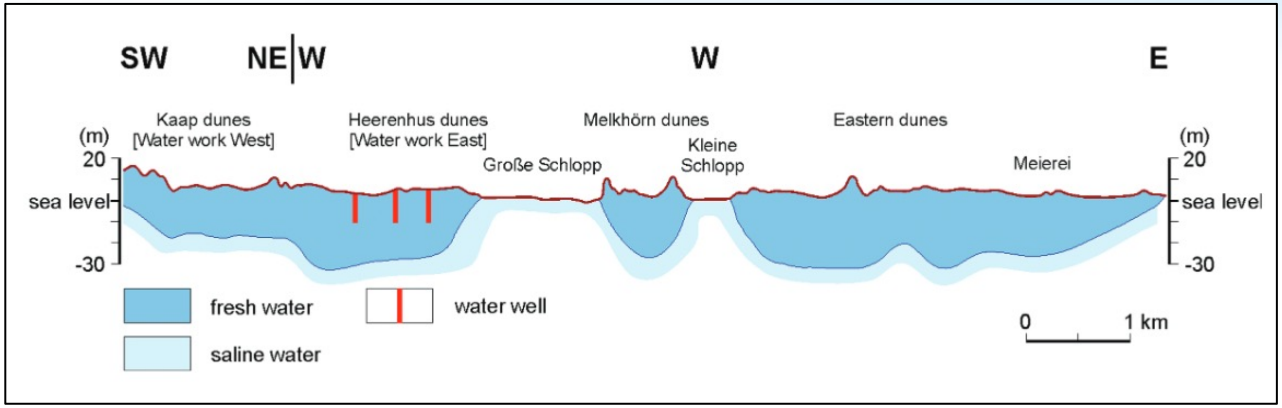
*Oberle et al. (2017)*

# Influence Factors

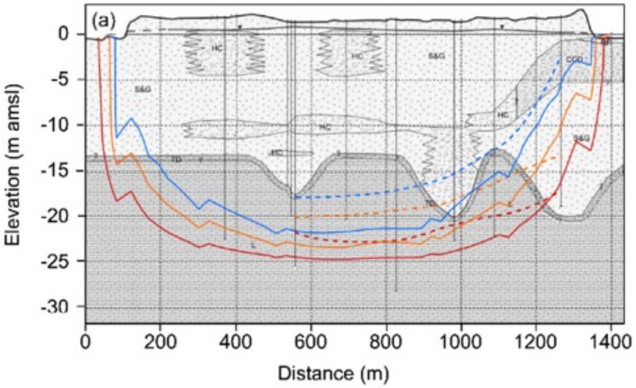


- Geometry: size and shape (circular island, barrier island, annulus segment island, elliptical island)
- Geology: hydraulic conductivity, aquifer heterogeneity
- Recharge and evaporation: temporal and spatial variation
- Oceanic boundary: tides, sea-level rise, storm surge
- Human activities: land reclamation, pumping, contaminant release

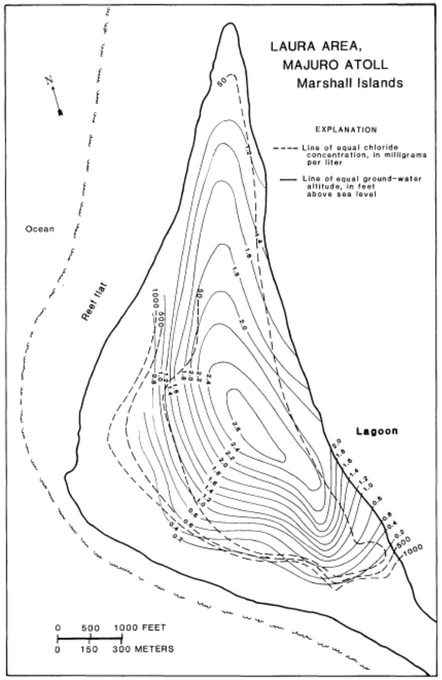
# Freshwater Lens



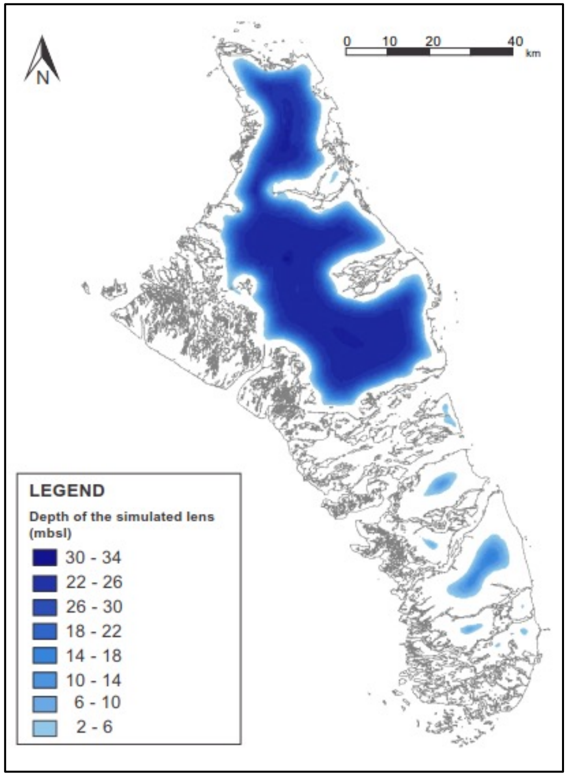
Langeoog Island, Germany (Streif,1990)



Bonriki Island, Kiribati (Post et al.,2018)

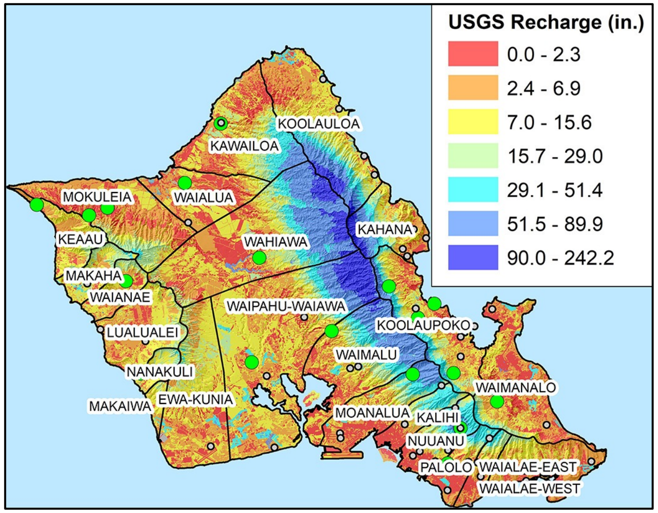


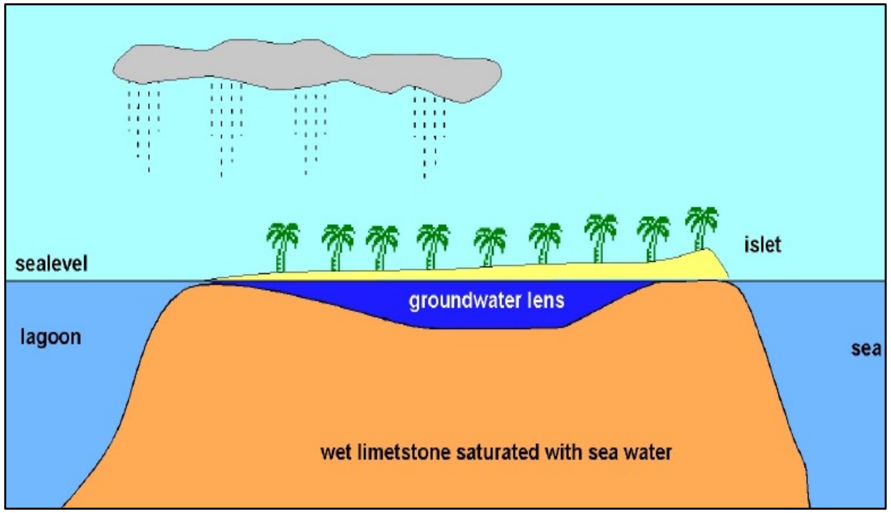
Majuro Atoll, Marshall Islands (Huxel,1973)



Andros Island, Bahamas (Holding and Allen,2015)

# Freshwater Lens Thickness



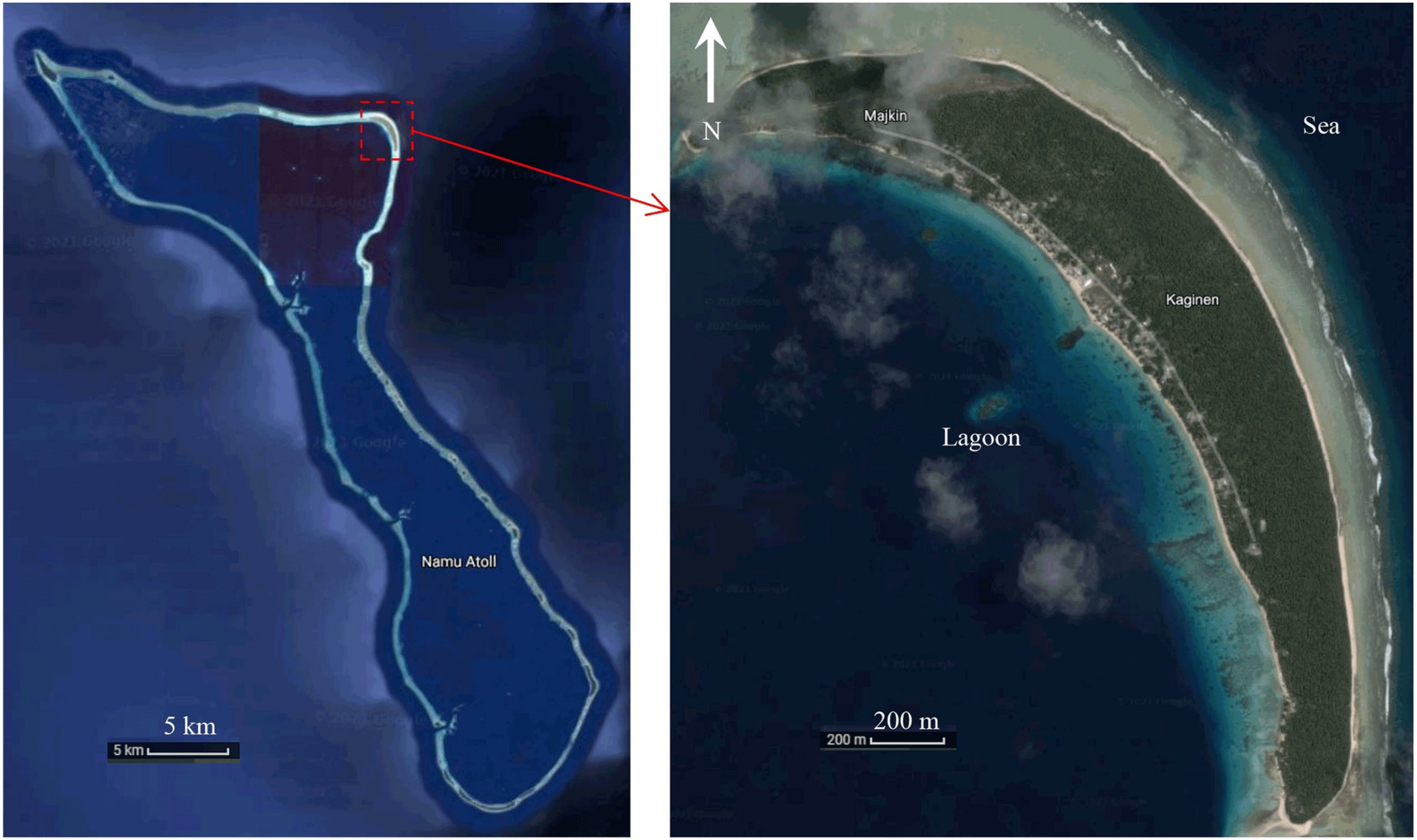


- The largest freshwater lens thickness is 304.8 m in Hawaii
- For small low-lying islands, the freshwater lens thickness is only several meters
- For some small islands, the freshwater lens only occurs in rain seasons and disappears in dry seasons

# Research Topics

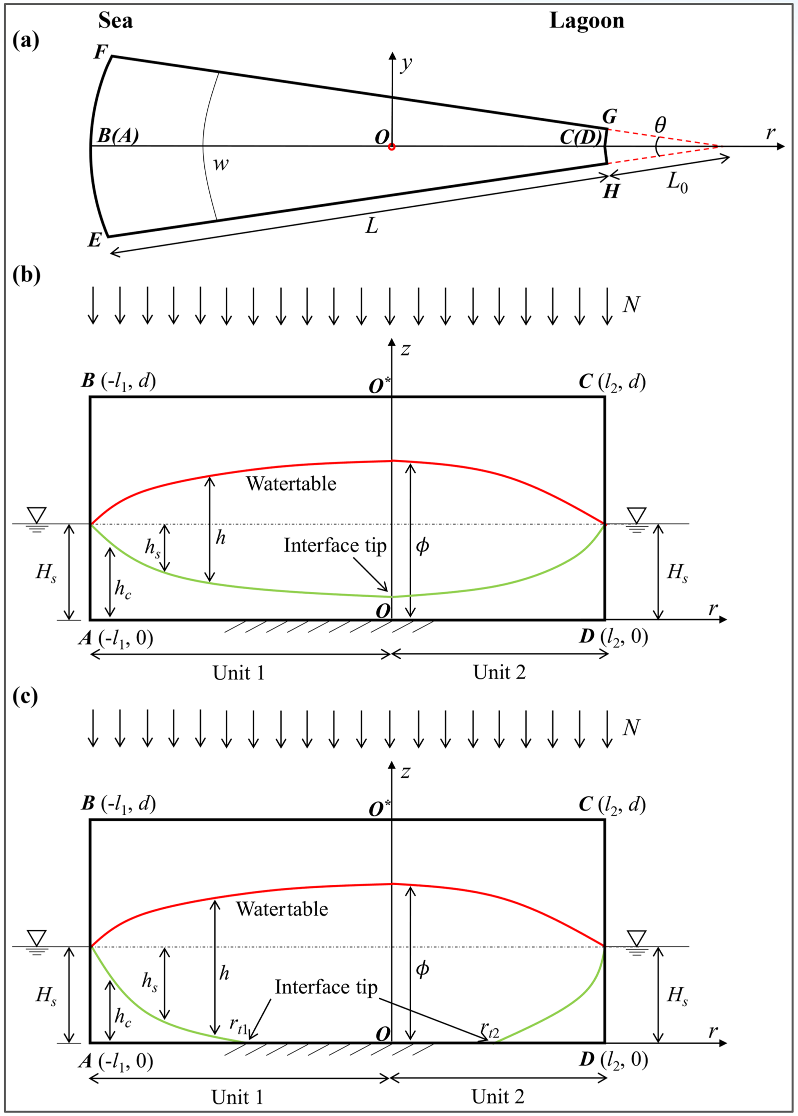
- Geometry effect (annulus segment and elliptical islands)
- Temporal and spatial variation in recharge
- A new concept for improving fresh groundwater storage and maximizing the well pumping rate
- Storm surge effect

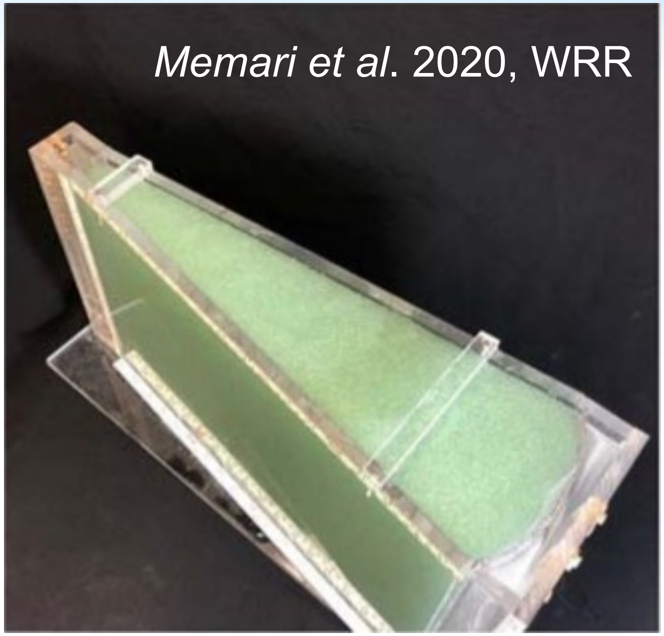
# Annulus Segment Island



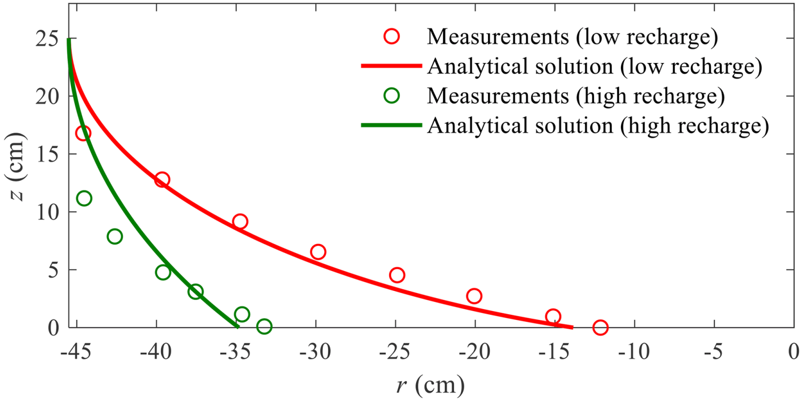
**Namu Atoll, Marshall Islands**

# Annulus Segment Island



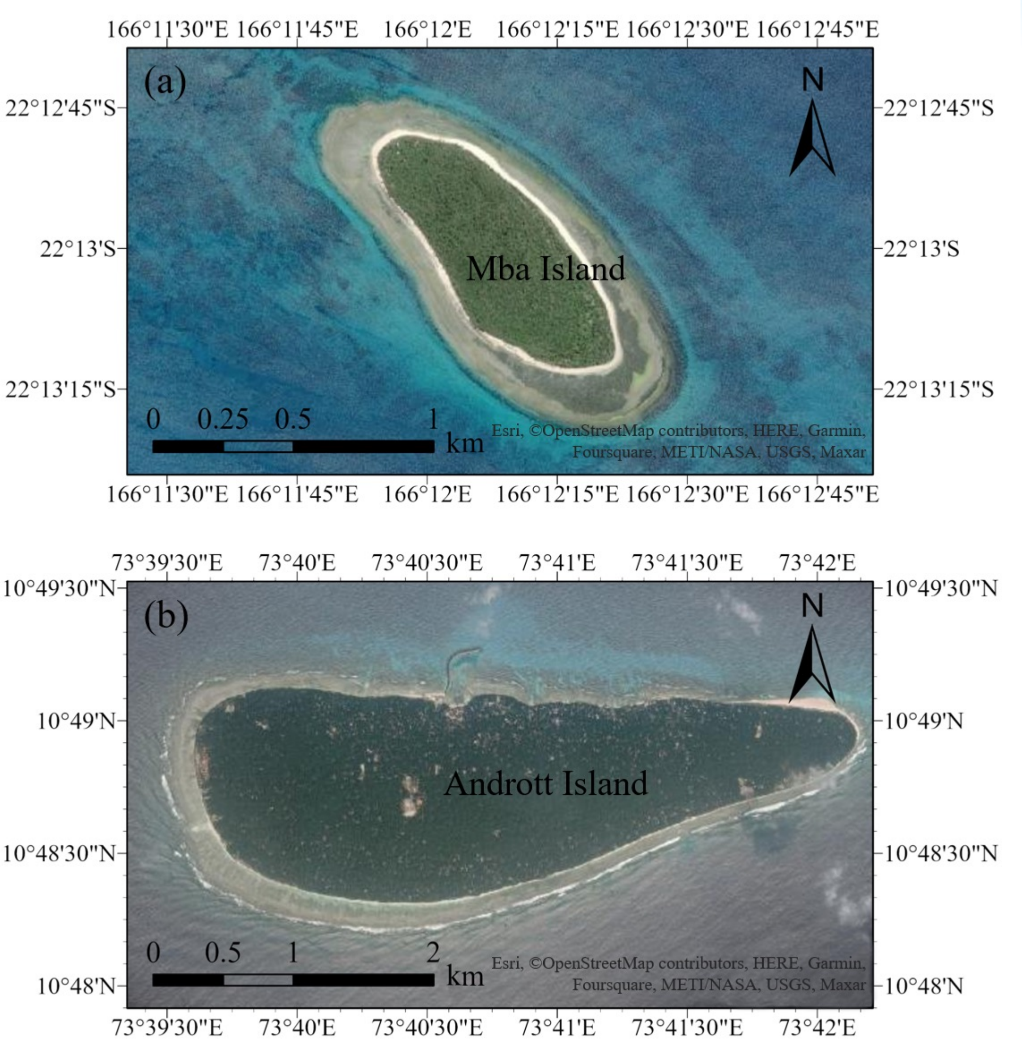


*Memari et al.* 2020, WRR



*Luo et al*., 2021, HESS

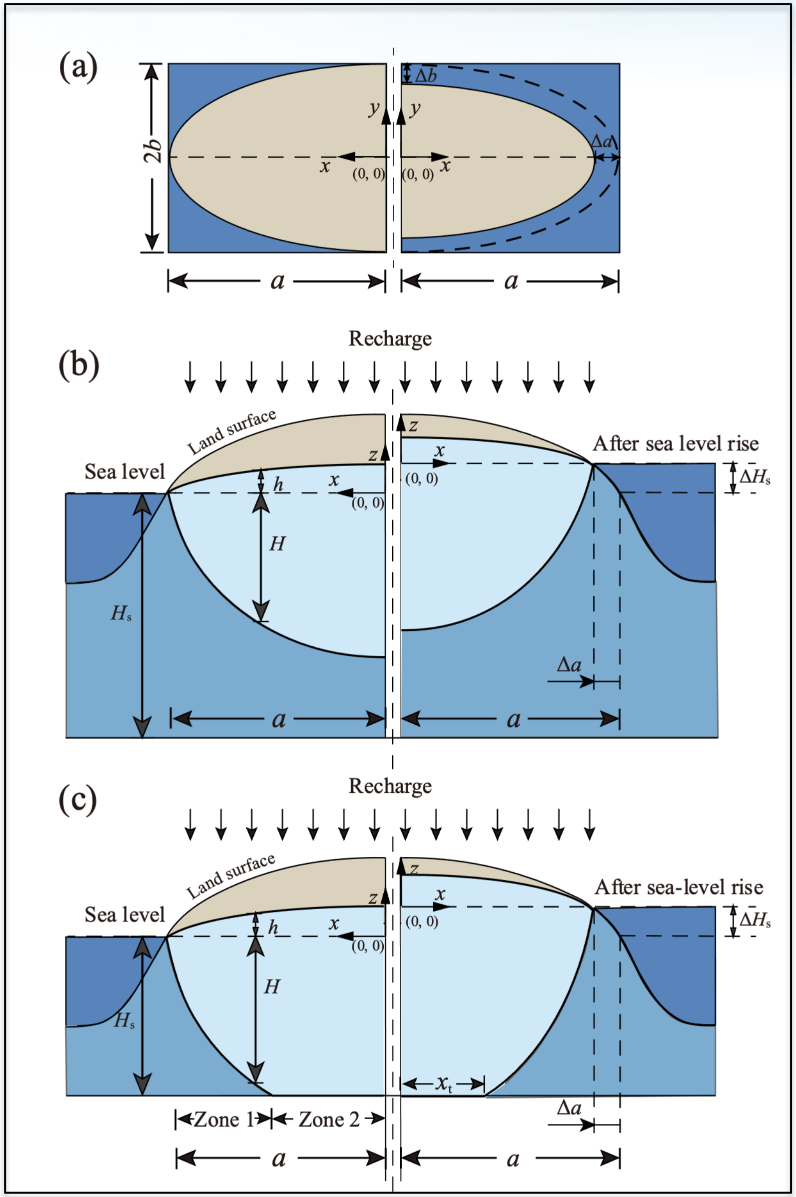
# Elliptical Island

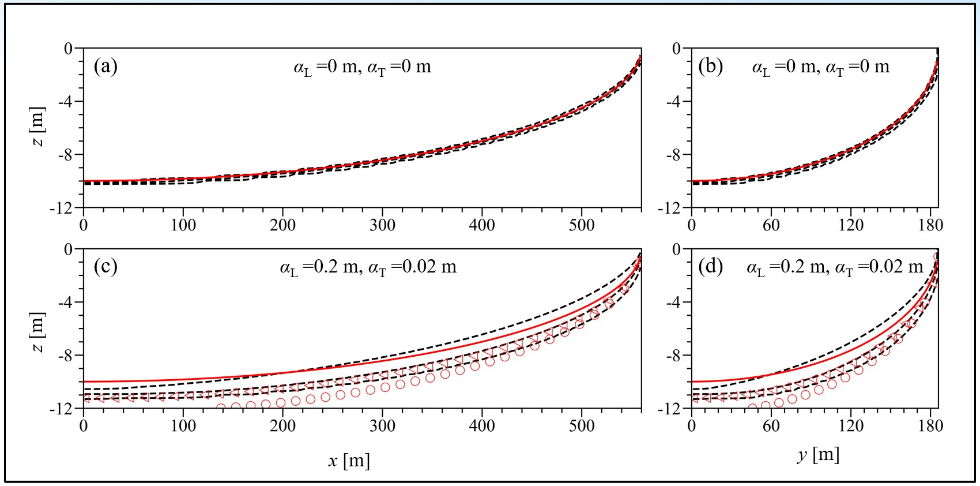


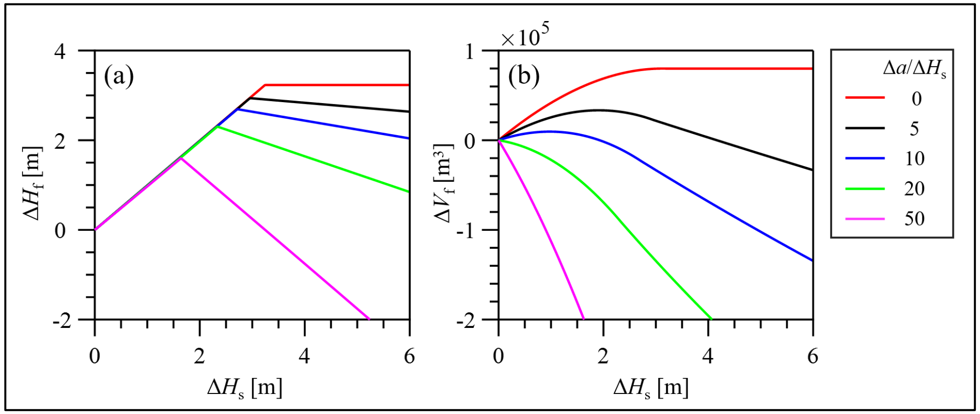
Esri, ©OpenStreetMap contributors, HERE, Garmin, Foursquare, METI/NASA, USGS, Maxar

Esri, ©OpenStreetMap contributors, HERE, Garmin, Foursquare, METI/NASA, USGS, Maxar

# Elliptical Island



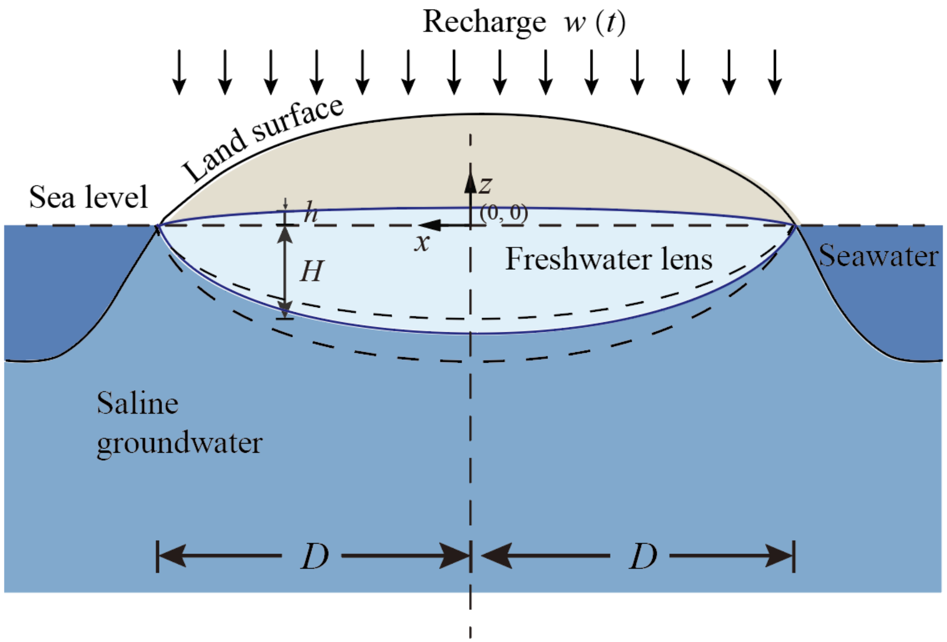




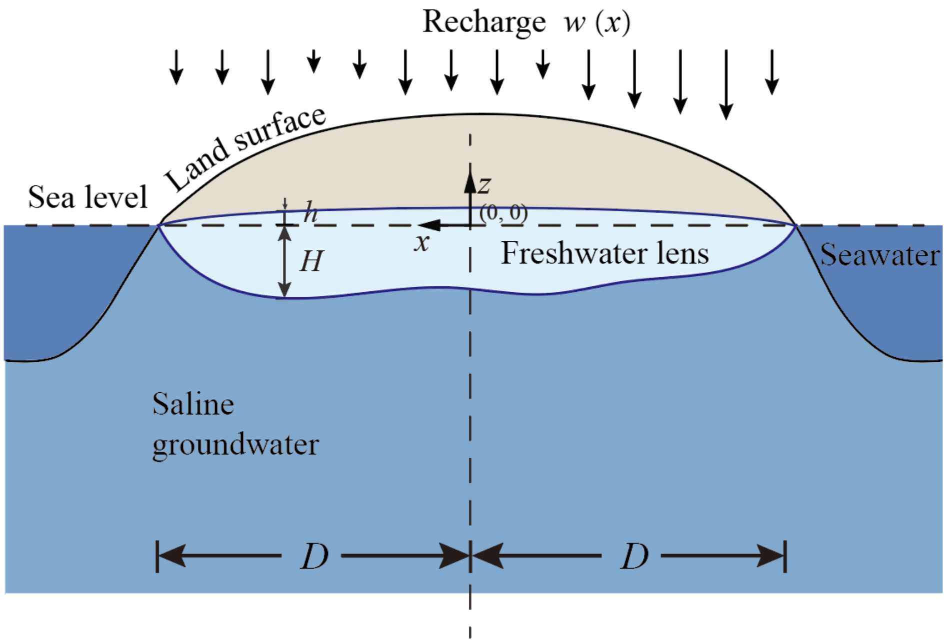
**Changes in FWL due to SLR and LSI**

*Yan et al.*, 2023, under review

# Temporal and Spatial Variation in Recharge



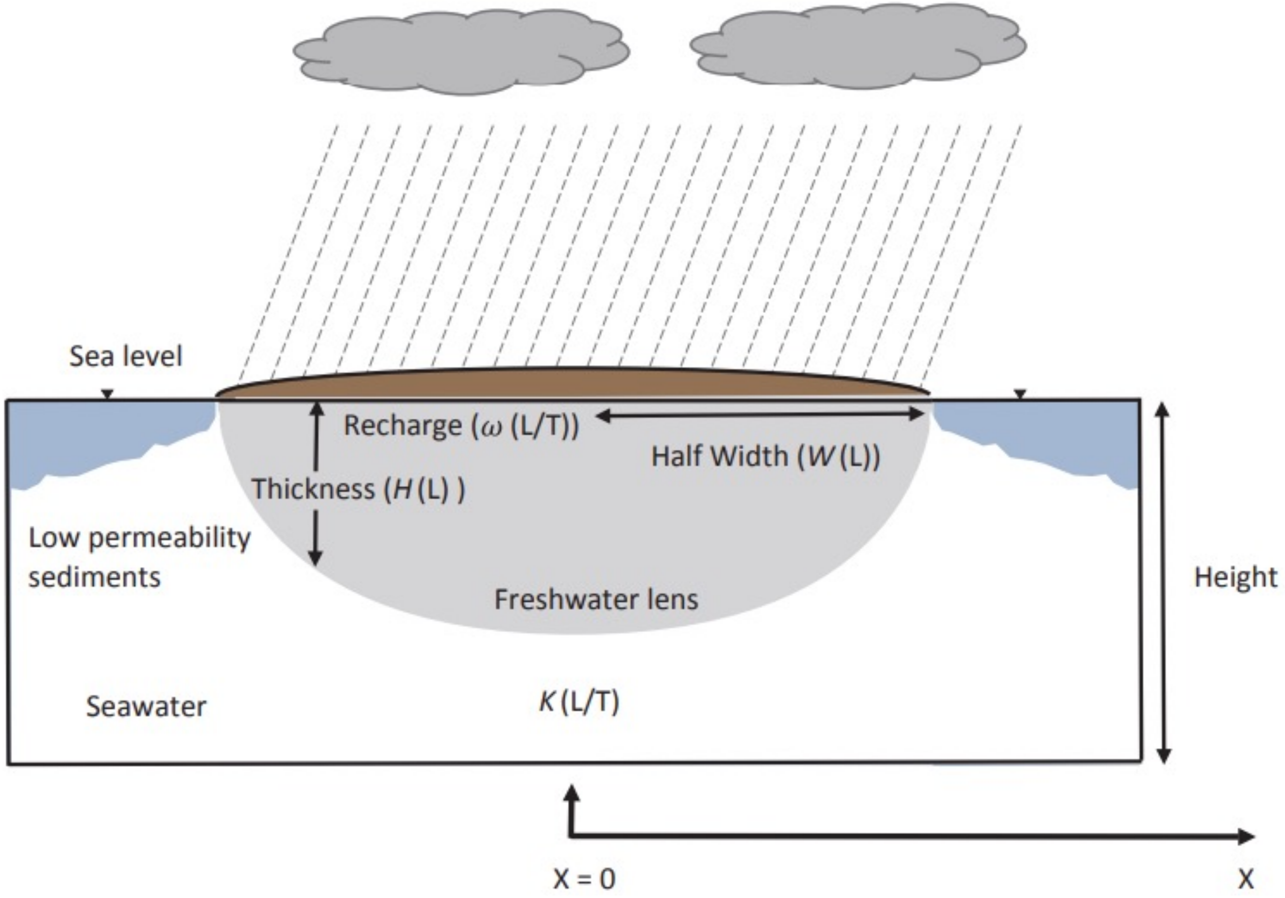
**Temporal Variation in Recharge**



**Spatial Variation in Recharge**

# Temporal Variation in Recharge

## Conceptual Model



# Temporal Variation in Recharge

## Approximate analytical solution

Normalization

$$H^* = \frac{H}{W}, w^* = \frac{w}{K}, \overline{H}^* = \frac{\overline{H}}{W}, t^* = \frac{Kt}{W\varepsilon}, x^* = \frac{x}{K}$$

Hantush Solution

$$H^{*2}\left(x^*, t^*\right) = \frac{w^* \overline{H}^* t^*}{1+\delta}\left[S^*\left(n^*\right) + S^*\left(\overline{n}^*\right)\right]$$

$$n^* = \frac{1+x^*}{\sqrt{4\delta \overline{H}^* t^*}}, \overline{n}^* = \frac{1-x^*}{\sqrt{4\delta \overline{H}^* t^*}}, S^*\left(\alpha\right) = \int_0^1 erf\left(\frac{\alpha}{\sqrt{\tau}}\right) d\tau$$

A linear convolution for
a time-dependent recharge
rate

$$H^{*2}\left(x^*, \ t^*\right) = \int_{-\infty}^{+\infty} w^*\left(\tau\right) h\left(x^*, t^* - \tau\right) d\tau$$

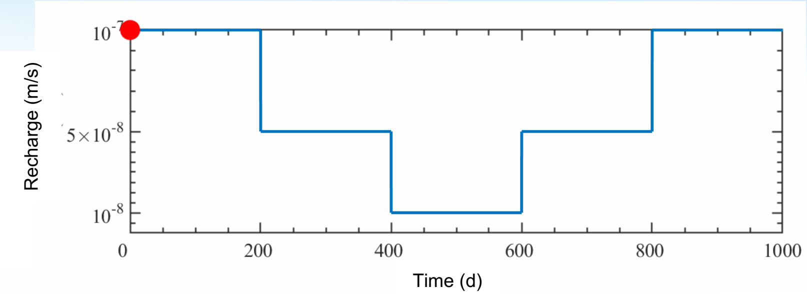
$$h\left(x^*, t^*\right) = \frac{w^* \overline{H}^*}{1+\delta}\left[S^*\left(n^*\right) + S^*\left(\overline{n}^*\right)\right] + \frac{w^* \overline{H}^* t^*}{1+\delta}\left[\frac{\partial S^*\left(n^*\right)}{\partial t^*} + \frac{\partial S^*\left(\overline{n}^*\right)}{\partial t^*}\right]$$

$$\frac{\partial S^*\left(n^*\right)}{\partial t^*} = -\frac{1+x^*}{4\sqrt{\delta \overline{H}^2}} t^{*-\frac{3}{2}} \int_0^1 \frac{2}{\sqrt{\pi\tau}} \exp\left(-\frac{n^{*2}}{\tau}\right) d\tau$$

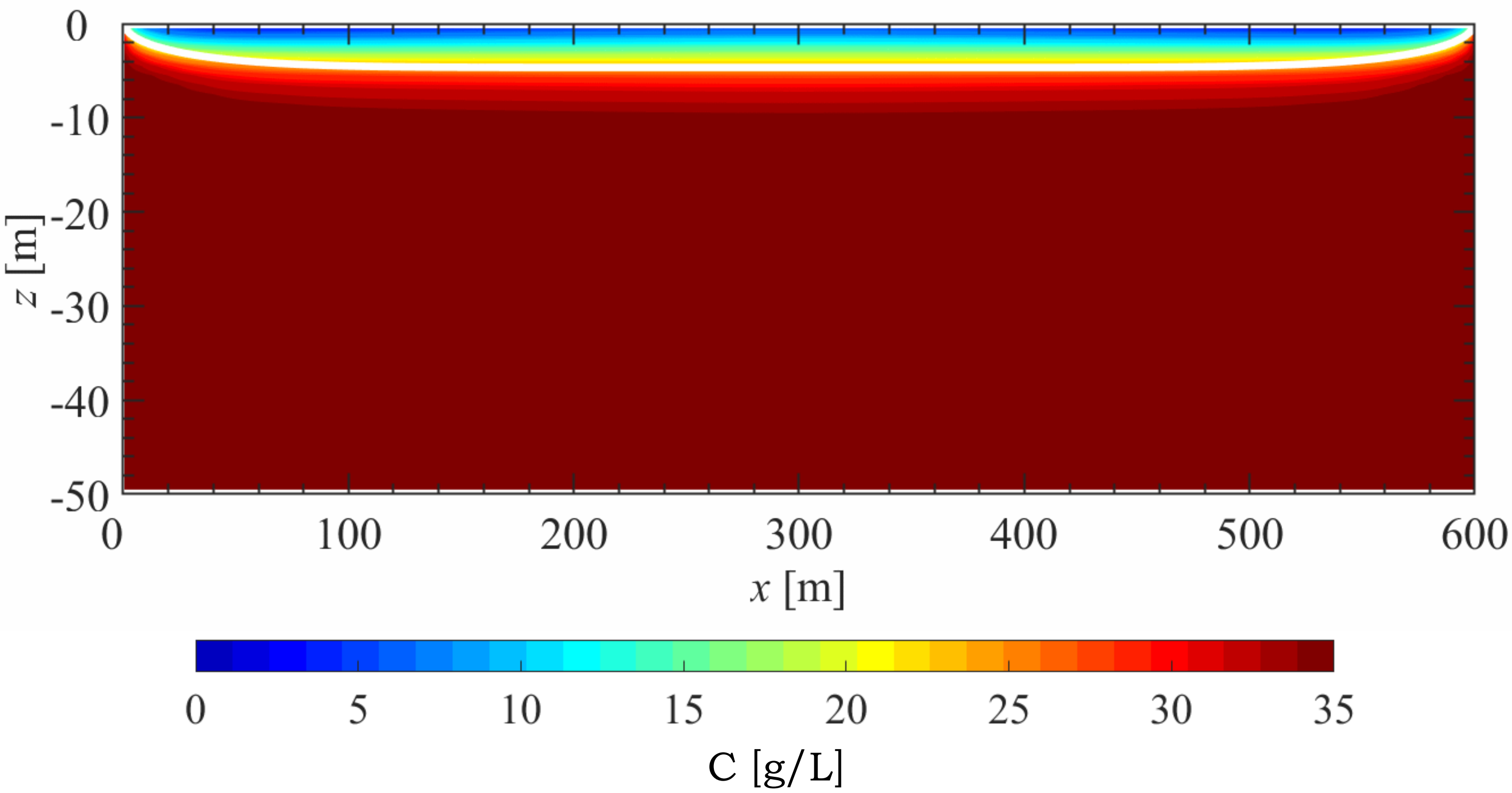
$$\frac{\partial S^*\left(\overline{n}^*\right)}{\partial t^*} = -\frac{1-x^*}{4\sqrt{\delta \overline{H}^2}} t^{*-\frac{3}{2}} \int_0^1 \frac{2}{\sqrt{\pi\tau}} \exp\left(-\frac{\overline{n}^{*2}}{\tau}\right) d\tau$$

*Tang et al.*, 2020, JH

# Temporal Variation in Recharge

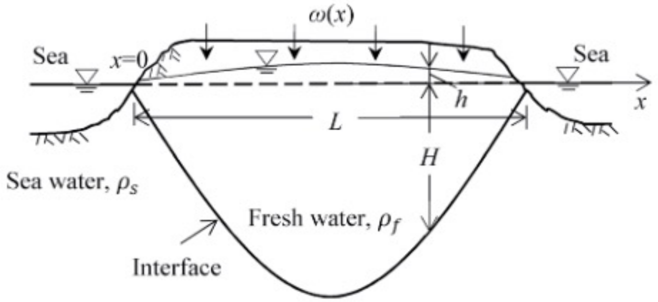


$t = 40$ d



# Spatial Variation in Recharge

Approximate analytical solution



$$\boxed{\frac{\partial q}{\partial x}=w(x)-\frac{\varepsilon\partial h}{\partial t}-\frac{\varepsilon\partial H}{\partial t}}$$

Darcy's Law ⇩ Ghyben-Herzberg Equation

$$\boxed{\frac{K\delta(\delta+1)}{2}\frac{\partial^2 H^2}{\partial x^2}+w(x)=\frac{\varepsilon(\delta+1)\partial H}{\partial t}}$$

⇩ Boussinesq linearization

$$\boxed{\bar{v}\frac{\partial^2 H^2}{\partial x^2}+Q(x)=\frac{\partial H^2}{\partial t},\bar{v}=\frac{K\delta\bar{H}}{\varepsilon},Q(x)=\frac{2w(x)\bar{v}}{\delta(1+\delta)K}}$$

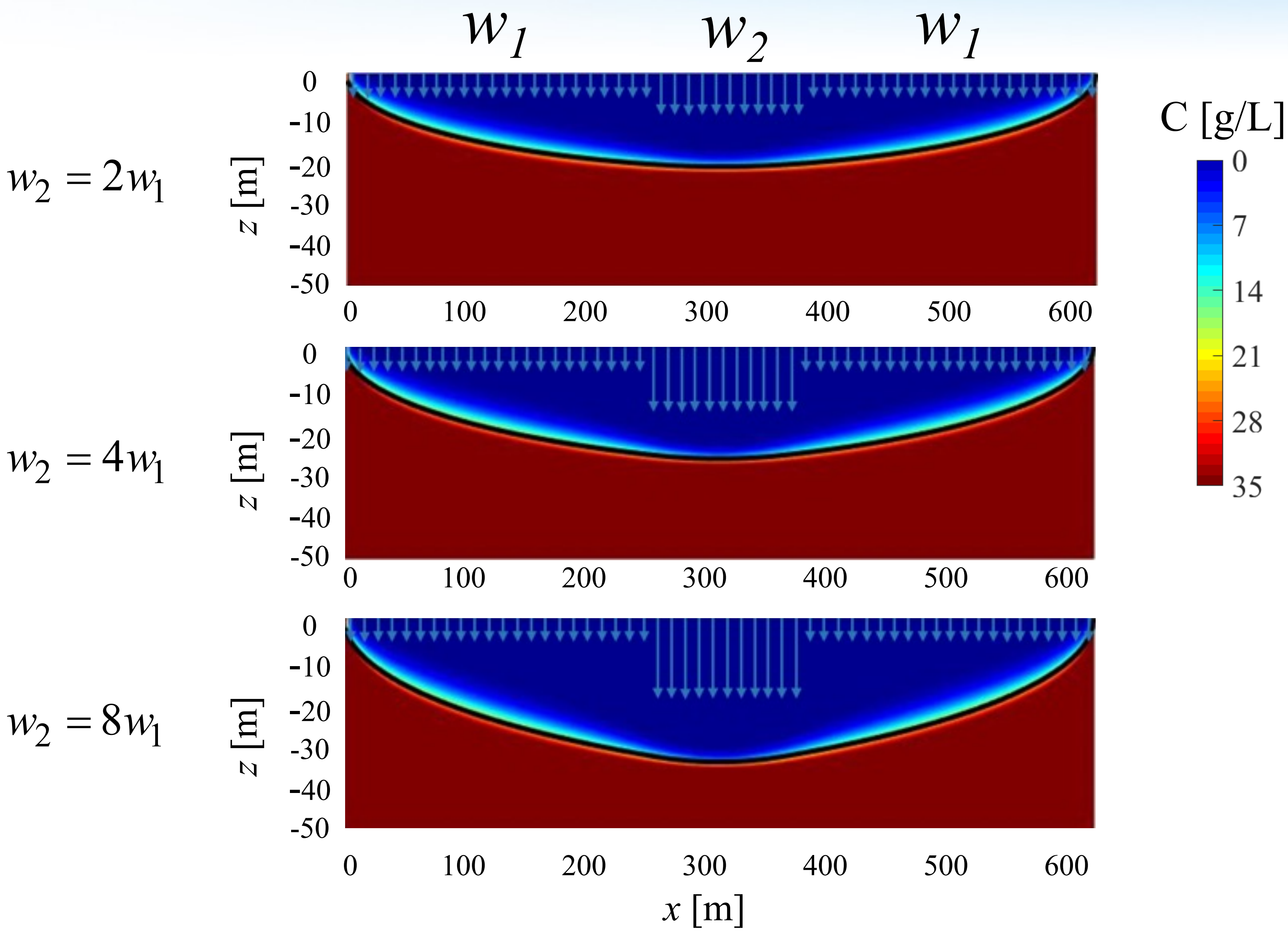
⇩ Fourier expansion

$$H^2(x,t)=\sum_{n=1}^{\infty}a_n(t)\phi_n(x)$$

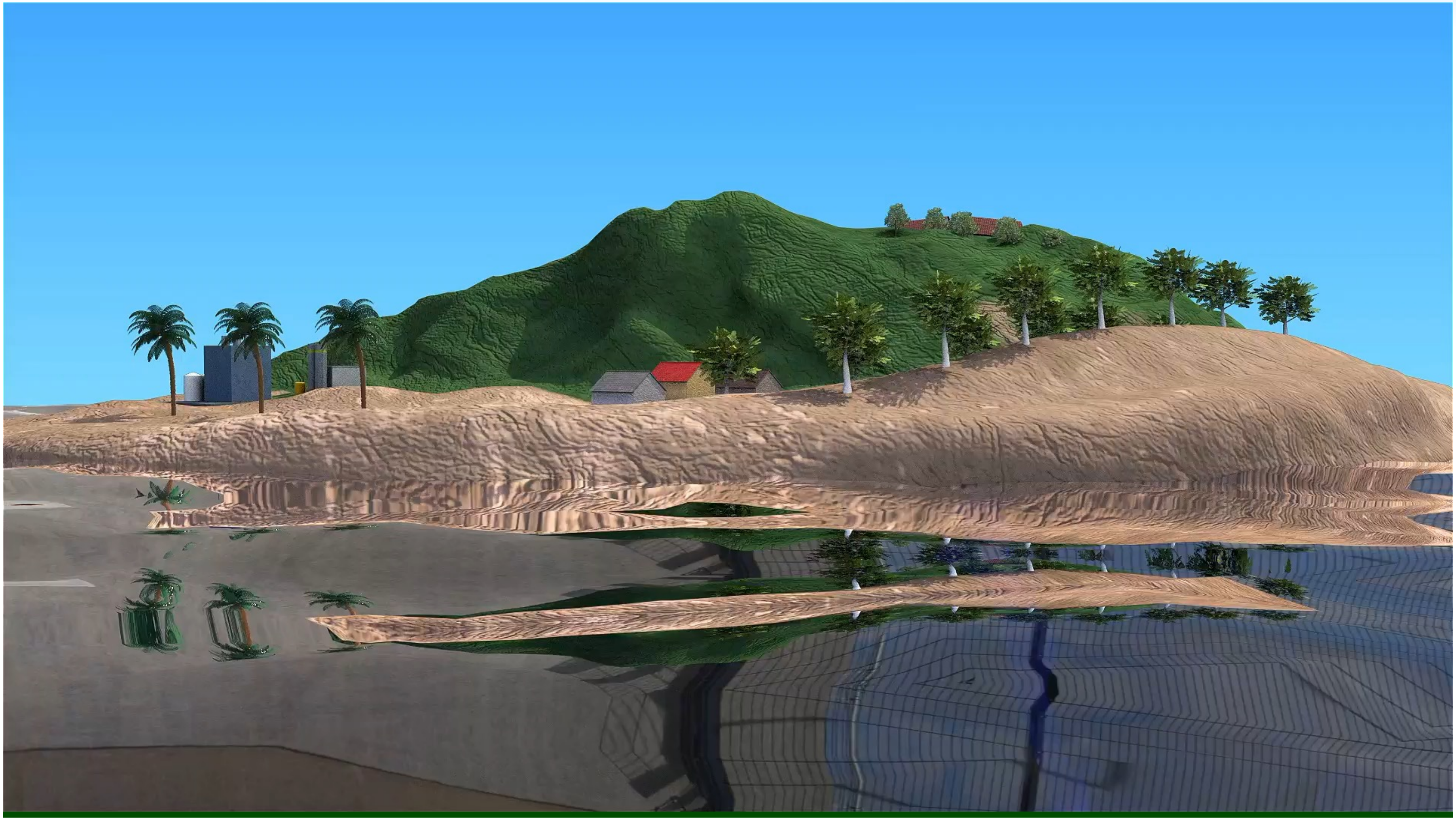
$$\phi_n(x)=\sin\frac{n\pi x}{L},a_n(t)=e^{-\lambda_n\bar{v}t}\int_0^t q_n e^{\lambda_n\bar{v}\tau}d\tau,\quad \lambda_n=\left(\frac{n\pi}{L}\right)^2,q_n=\frac{2\int_0^L Q(x)\phi_n(x)dx}{L}$$

*Tang et al*. 2021, WRR

# Spatial Variation in Recharge

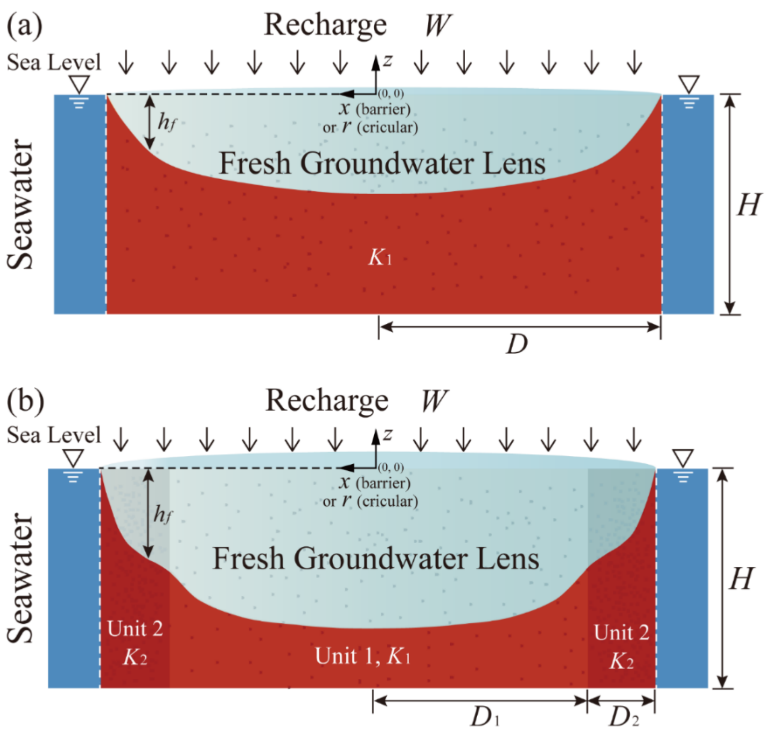


# A New Concept



# Fully Penetrating Barrier

## Conceptual model



## Assumption

☐ Uniform recharge

☐ Sharp-interface

☐ Ghyben-herzberg approximation

☐ Dupuit-Forchheimer assumption

## Darcy's law and Continuity equation

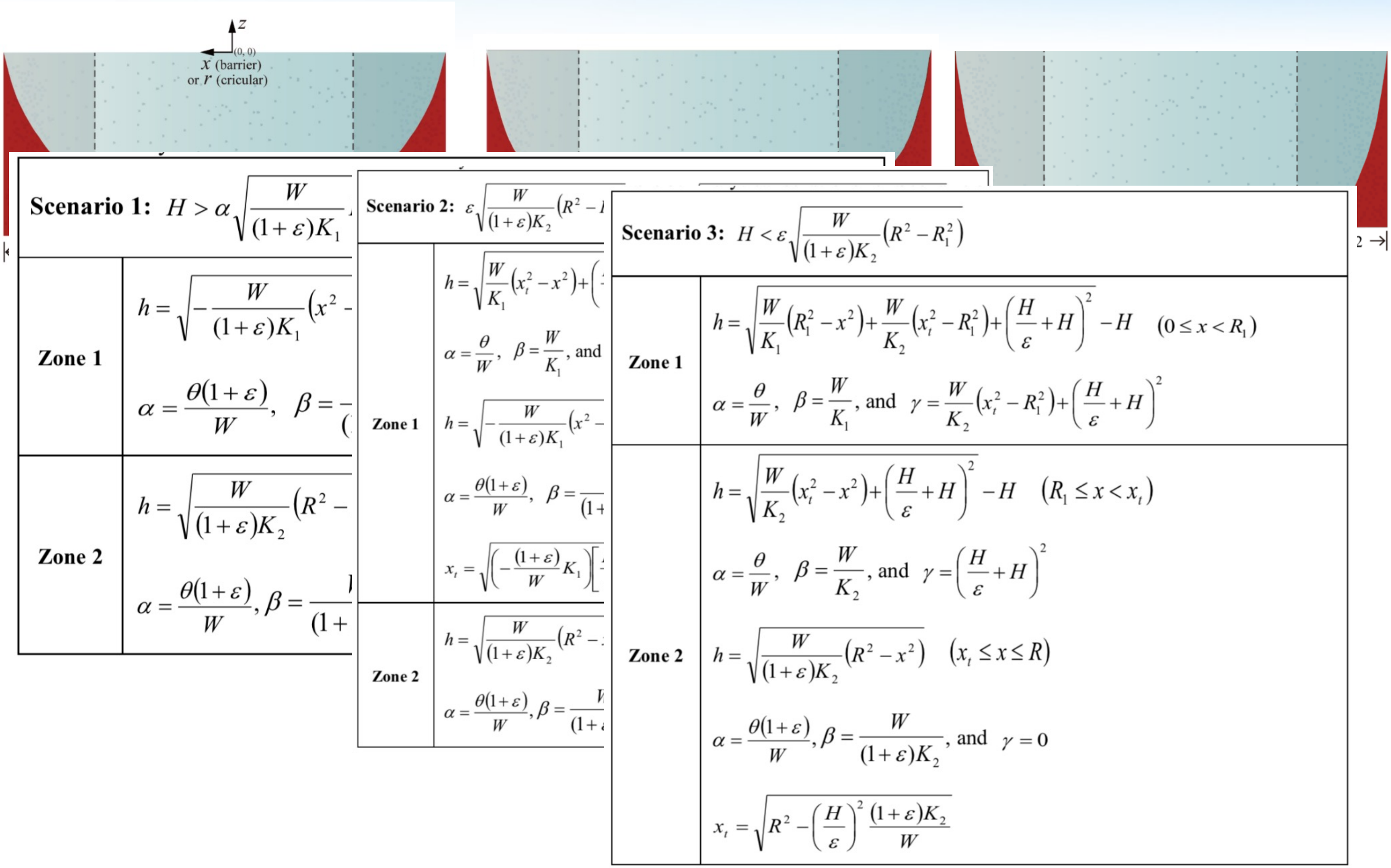
Strip island: $$\frac{d}{dx}\left[K(1+\varepsilon)h\frac{dh}{dx}\right]+W=0$$

Circular island: $$\frac{d}{dr}\left[K(1+\varepsilon)h\frac{dh}{dr}\right]+\frac{1}{r}\left[K(1+\varepsilon)h\frac{dh}{dr}\right]+W=0$$

| | |
|---|---|
| $h$ water table | $K$ hydraulic conductivity |
| $W$ recharge rate | $\varepsilon$ Ghyben-herzberg ratio |

*Lu et al*. 2019, WRR

# Analytical Solutions



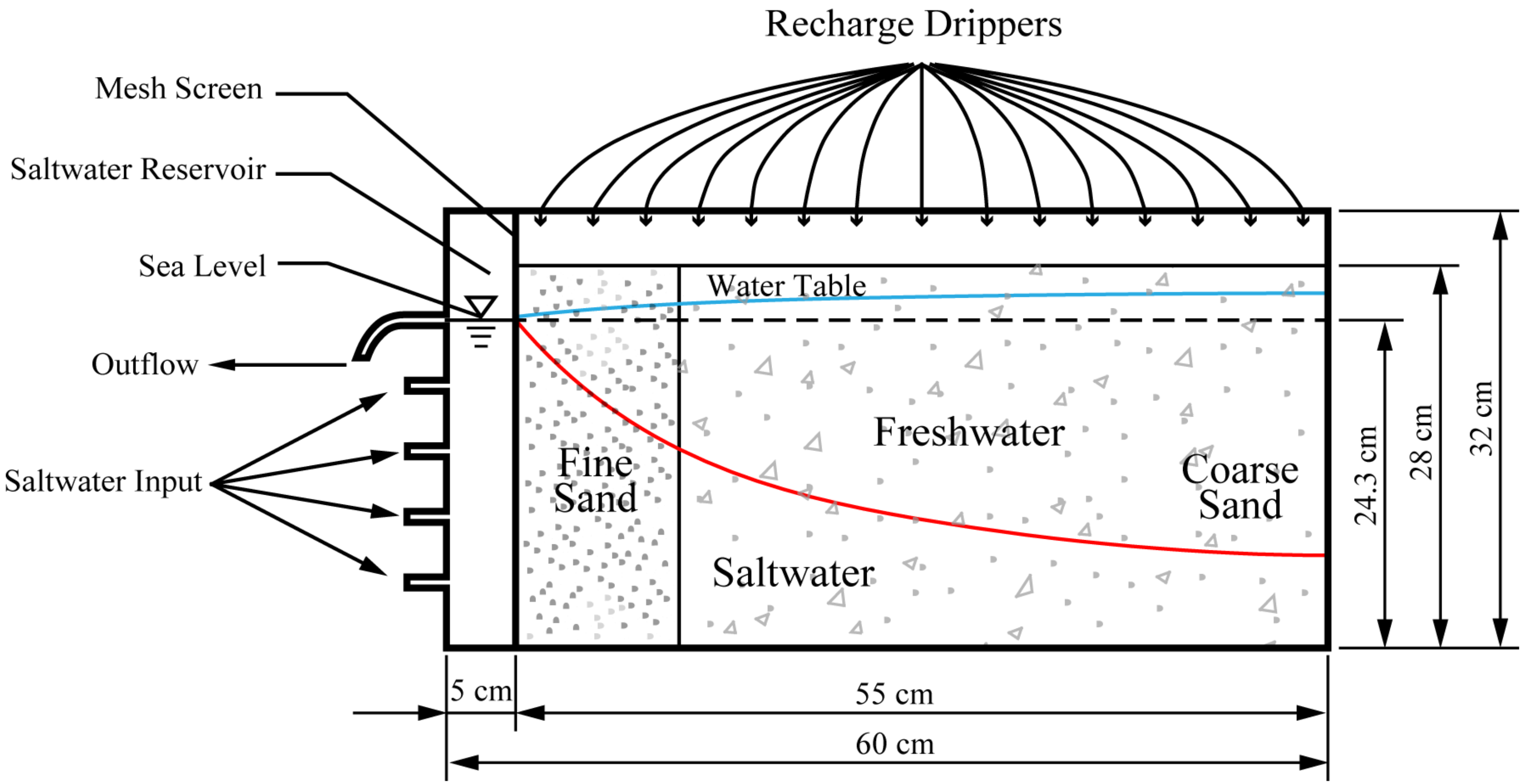
| **Scenario 1:** $H > \alpha\sqrt{\frac{W}{(1+\varepsilon)K_1}}$ | |
|---|---|
| **Zone 1** | $h = \sqrt{-\frac{W}{(1+\varepsilon)K_1}(x^2 -}$<br>$\alpha = \frac{\theta(1+\varepsilon)}{W}$, $\beta = -$ |
| **Zone 2** | $h = \sqrt{\frac{W}{(1+\varepsilon)K_2}(R^2 -}$<br>$\alpha = \frac{\theta(1+\varepsilon)}{W}, \beta = \frac{}{(1+}$ |

| **Scenario 2:** $\varepsilon\sqrt{\frac{W}{(1+\varepsilon)K_2}(R^2 -}$ | |
|---|---|
| **Zone 1** | $h = \sqrt{\frac{W}{K_1}(x_t^2 - x^2) + (}$<br>$\alpha = \frac{\theta}{W}$, $\beta = \frac{W}{K_1}$, and<br>$h = \sqrt{-\frac{W}{(1+\varepsilon)K_1}(x^2 -}$<br>$\alpha = \frac{\theta(1+\varepsilon)}{W}$, $\beta = \frac{}{(1+}$<br>$x_t = \sqrt{\left(-\frac{(1+\varepsilon)}{W}K_1\right)[}$ |
| **Zone 2** | $h = \sqrt{\frac{W}{(1+\varepsilon)K_2}(R^2 -}$<br>$\alpha = \frac{\theta(1+\varepsilon)}{W}, \beta = \frac{W}{(1+}$ |

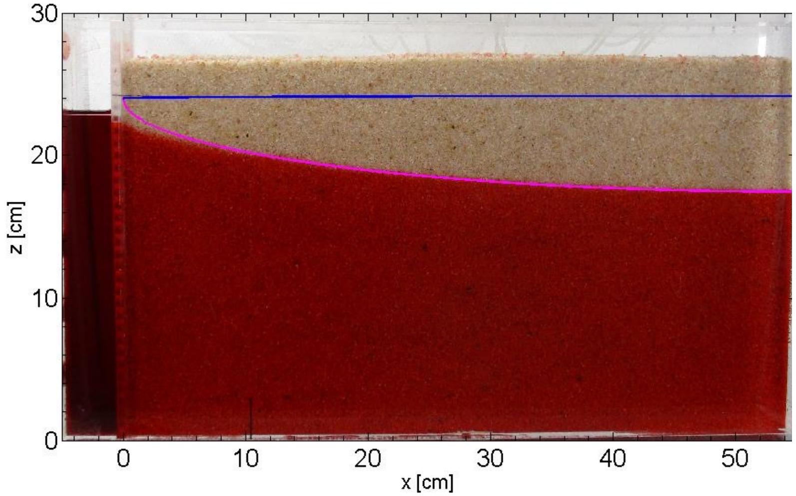
| **Scenario 3:** $H < \varepsilon\sqrt{\frac{W}{(1+\varepsilon)K_2}(R^2 - R_1^2)}$ | |
|---|---|
| **Zone 1** | $h = \sqrt{\frac{W}{K_1}(R_1^2 - x^2) + \frac{W}{K_2}(x_t^2 - R_1^2) + \left(\frac{H}{\varepsilon} + H\right)^2} - H \quad (0 \le x < R_1)$<br>$\alpha = \frac{\theta}{W}$, $\beta = \frac{W}{K_1}$, and $\gamma = \frac{W}{K_2}(x_t^2 - R_1^2) + \left(\frac{H}{\varepsilon} + H\right)^2$ |
| **Zone 2** | $h = \sqrt{\frac{W}{K_2}(x_t^2 - x^2) + \left(\frac{H}{\varepsilon} + H\right)^2} - H \quad (R_1 \le x < x_t)$<br>$\alpha = \frac{\theta}{W}$, $\beta = \frac{W}{K_2}$, and $\gamma = \left(\frac{H}{\varepsilon} + H\right)^2$<br>$h = \sqrt{\frac{W}{(1+\varepsilon)K_2}(R^2 - x^2)} \quad (x_t \le x \le R)$<br>$\alpha = \frac{\theta(1+\varepsilon)}{W}, \beta = \frac{W}{(1+\varepsilon)K_2}$, and $\gamma = 0$<br>$x_t = \sqrt{R^2 - \left(\frac{H}{\varepsilon}\right)^2 \frac{(1+\varepsilon)K_2}{W}}$ |

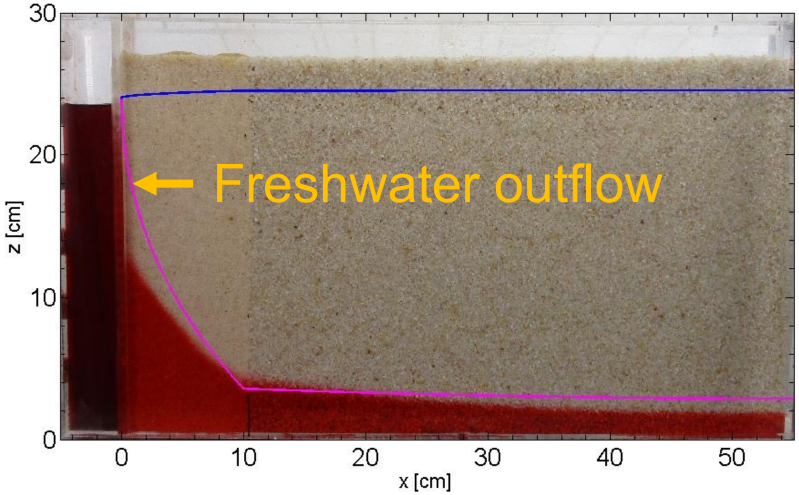
*Lu et al*. 2019, WRR

# Laboratory Setup



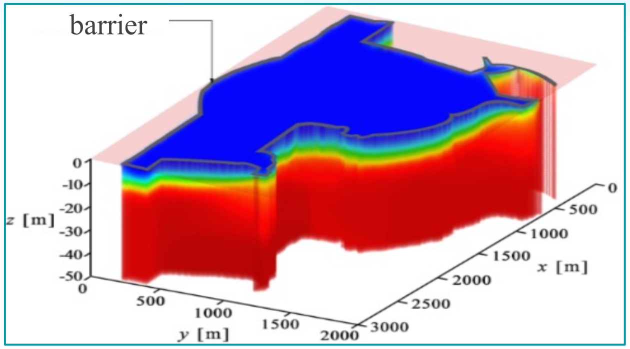
# Experimental Vs. Analytical Solution

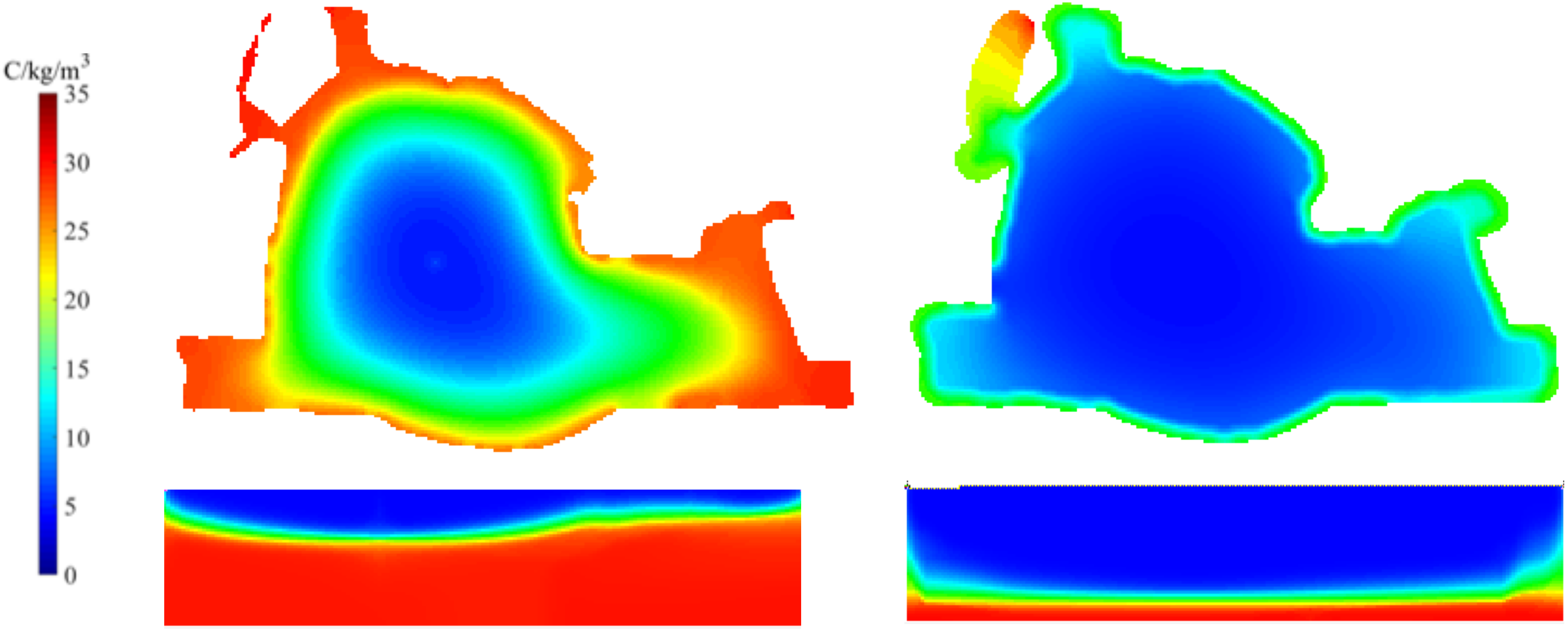



Freshwater outflow

| Parameter | Symbol | Value | Parameter | Symbol | Value |
|---|---|---|---|---|---|
| Aquifer depth | $H$ | 24.5 cm | Half-width of the island | $D$ | 55 cm |
| Recharge rate | $W$ | 0.18 cm/min | Hydraulic conductivity of original island | $K_1$ | 500 cm/min |
| Barrier width | $D_2$ | 10 cm | Hydraulic Conductivity of barrier | $K_2$ | 12.4 cm/min |

# Case Study - Yongxing Island



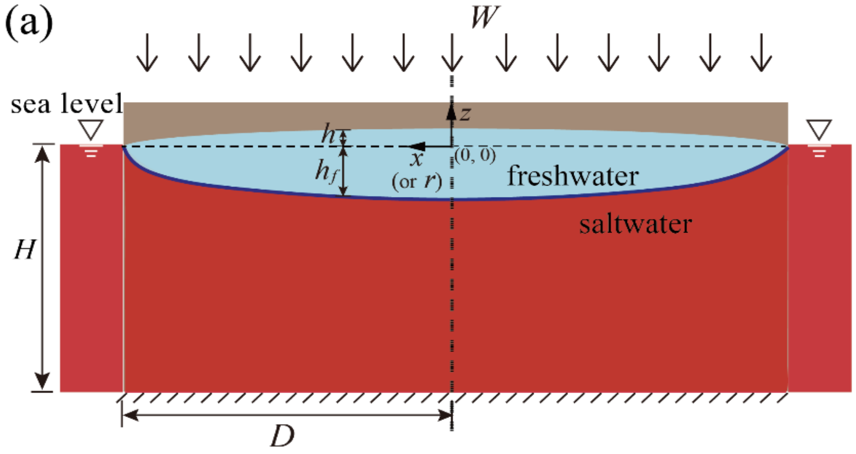


***Before***

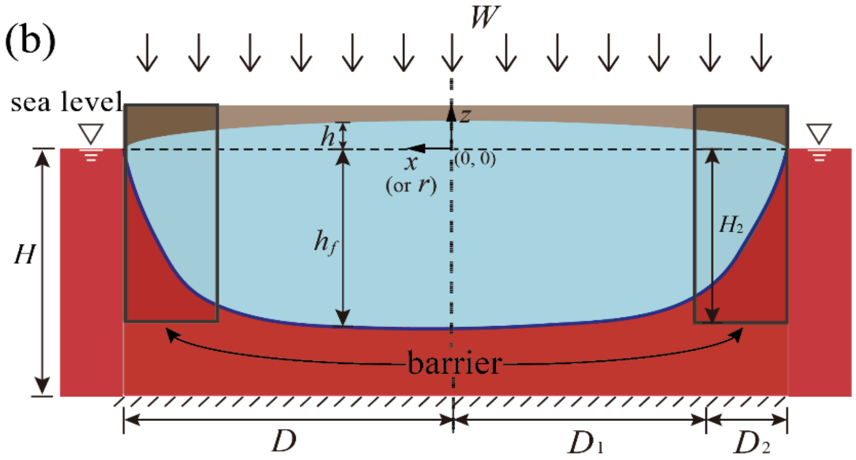
***After***

## ***Q1: Can we use a partially penetrating low-permeability barrier to enhance fresh groundwater storage?***

# Conceptual Model

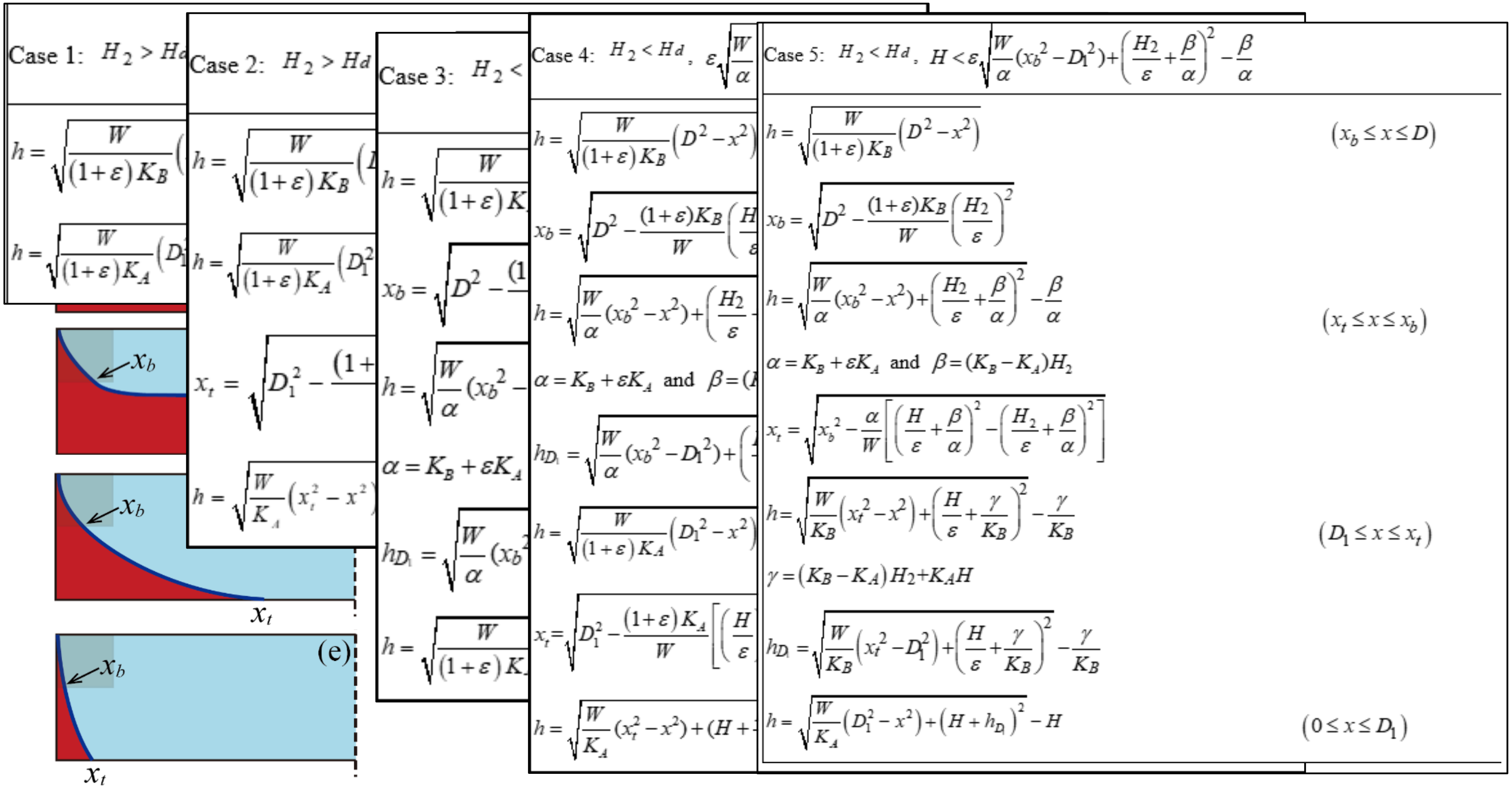


Original homogeneous island



Partially penetrating barrier

# Analytical Solution



Case 5: $H_2 < H_d$, $H < \varepsilon\sqrt{\frac{W}{\alpha}(x_b{}^2 - D_1{}^2) + \left(\frac{H_2}{\varepsilon} + \frac{\beta}{\alpha}\right)^2} - \frac{\beta}{\alpha}$

$$h = \sqrt{\frac{W}{(1+\varepsilon)K_B}\left(D^2 - x^2\right)} \qquad \left(x_b \le x \le D\right)$$

$$x_b = \sqrt{D^2 - \frac{(1+\varepsilon)K_B}{W}\left(\frac{H_2}{\varepsilon}\right)^2}$$

$$h = \sqrt{\frac{W}{\alpha}(x_b{}^2 - x^2) + \left(\frac{H_2}{\varepsilon} + \frac{\beta}{\alpha}\right)^2} - \frac{\beta}{\alpha} \qquad \left(x_t \le x \le x_b\right)$$

$\alpha = K_B + \varepsilon K_A$ and $\beta = (K_B - K_A)H_2$

$$x_t = \sqrt{x_b{}^2 - \frac{\alpha}{W}\left[\left(\frac{H}{\varepsilon} + \frac{\beta}{\alpha}\right)^2 - \left(\frac{H_2}{\varepsilon} + \frac{\beta}{\alpha}\right)^2\right]}$$

$$h = \sqrt{\frac{W}{K_B}\left(x_t^2 - x^2\right) + \left(\frac{H}{\varepsilon} + \frac{\gamma}{K_B}\right)^2} - \frac{\gamma}{K_B} \qquad \left(D_1 \le x \le x_t\right)$$

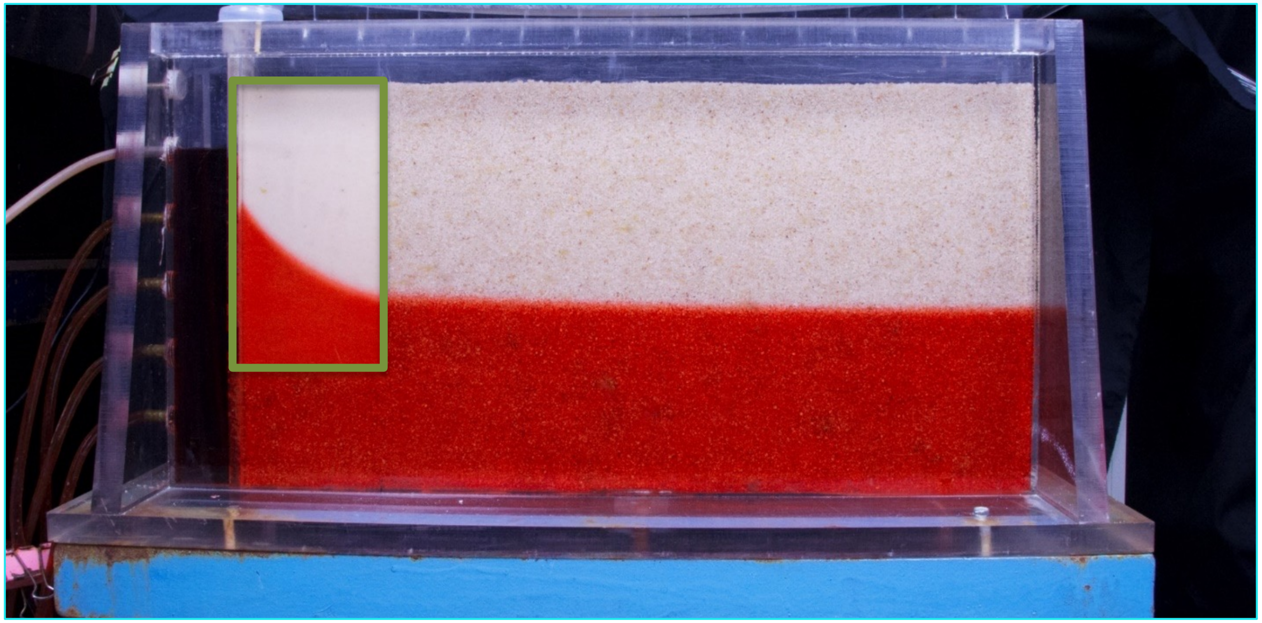
$\gamma = \left(K_B - K_A\right)H_2 + K_A H$

$$h_{D_1} = \sqrt{\frac{W}{K_B}\left(x_t^2 - D_1^2\right) + \left(\frac{H}{\varepsilon} + \frac{\gamma}{K_B}\right)^2} - \frac{\gamma}{K_B}$$

$$h = \sqrt{\frac{W}{K_A}\left(D_1^2 - x^2\right) + \left(H + h_{D_1}\right)^2} - H \qquad \left(0 \le x \le D_1\right)$$

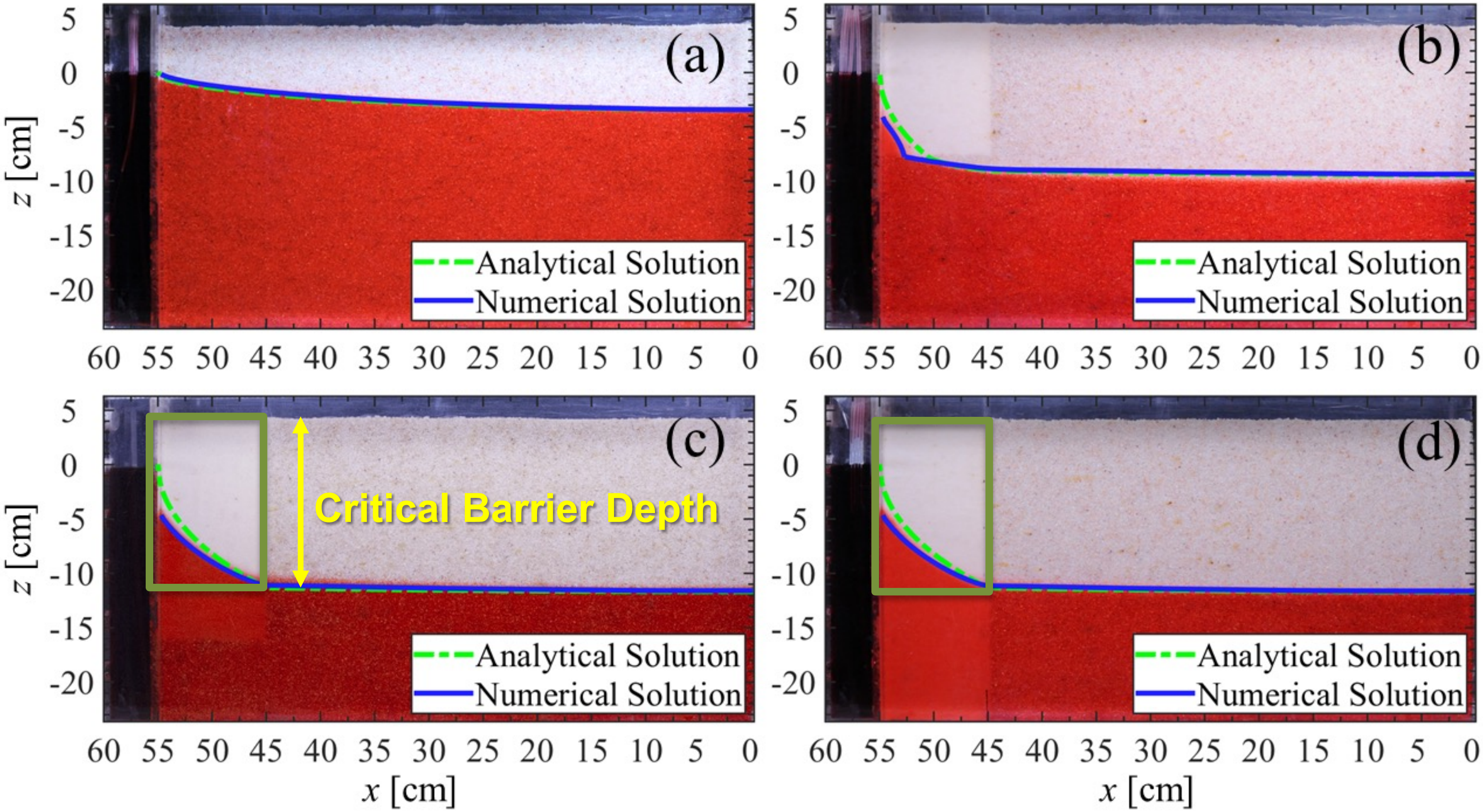
*Yan et al., 2021,* WRR

# Laboratory Experiment



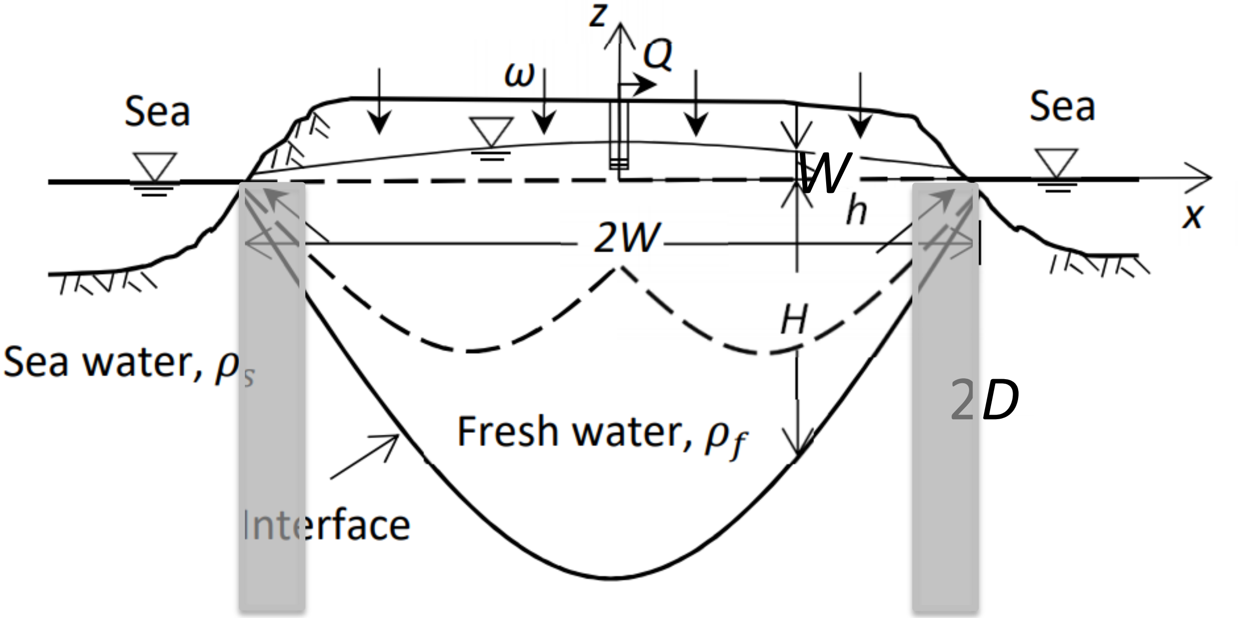
| Parameter | Symbol | Value | Parameter | Symbol | Value |
|---|---|---|---|---|---|
| Aquifer depth | $H$ | 23.7 cm | Recharge rate | $W$ | 0.036 cm/min |
| Barrier width | $D_2$ | 10 cm | Half-width of the island | $D$ | 55 cm |
| Barrier depth (under sea level) | $H_2$ | a) 0 cm | Hydraulic conductivity of original island | $K_1$ | 360 cm/min |
| | | b) 8 cm | | | |
| | | c) 16 cm | Hydraulic Conductivity of barrier | $K_2$ | 11 cm/min |
| | | d) 23.7 cm | | | |

# Sensitivity Analysis



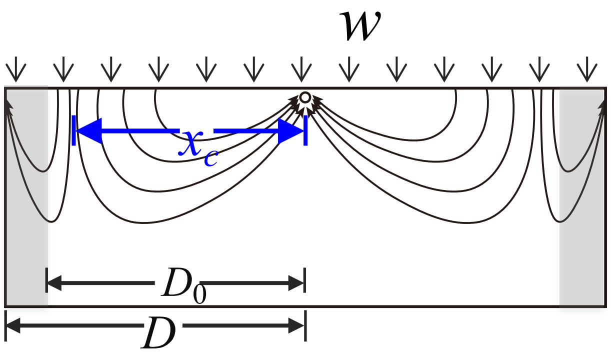
## *Q2: How much freshwater can be extracted from a freshwater lens of a small island?*

# Conceptual Model



Maximum pumping rate: under the steady state condition, the saltwater interface just reaches the bottom of the well screen.

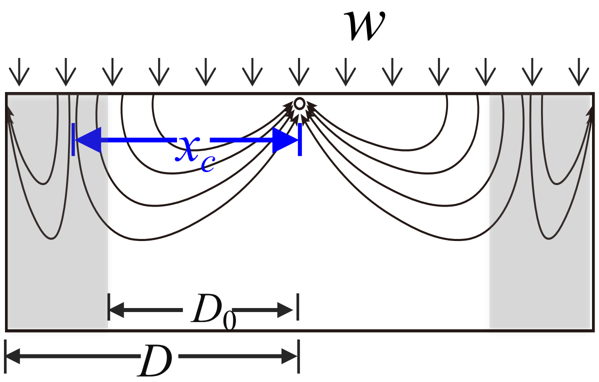
# Analytical Solutions



$$h^2=\frac{w}{K_b(1+\varepsilon)}\left[(D-x_c)^2-(x-x_c)^2\right]\qquad (D_0<x<D)$$

$$h^2=\frac{w}{K_0(1+\varepsilon)}\left[(D_0-x_c)^2-(x-x_c)^2\right]+\frac{w}{K_b(1+\varepsilon)}\left[(D-x_c)^2-(D_0-x_c)^2\right]\qquad (x_c<x<D_0)$$

$$h^2=\frac{w}{K_0(1+\varepsilon)}\left[(D_0-x_c)^2-(x_c-x)^2\right]+\frac{w}{K_b(1+\varepsilon)}\left[(D-x_c)^2-(D_0-x_c)^2\right]\qquad (0<x<x_c)$$



$$h^2=\frac{w}{K_b(1+\varepsilon)}\left[(D-x_c)^2-(x-x_c)^2\right]\qquad (x_c<x<D)$$

$$h^2=\frac{w}{K_b(1+\varepsilon)}\left[(D-x_c)^2-(x-x_c)^2\right]\qquad (D_0<x<x_c)$$

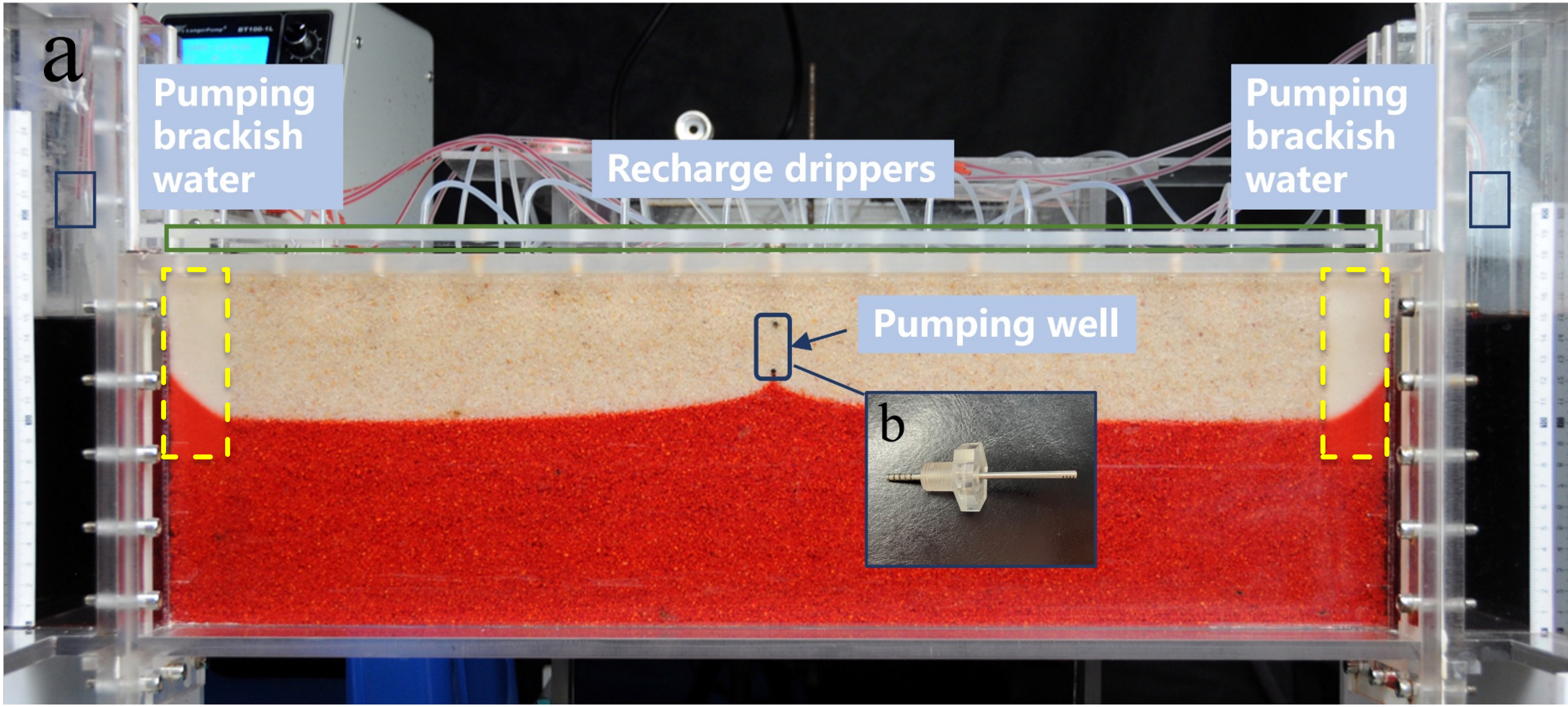
$$h^2=\frac{w}{K_0(1+\varepsilon)}\left[(D_0-x_c)^2-(x_c-x)^2\right]+\frac{w}{K_b(1+\varepsilon)}\left[(D-x_c)^2-(D_0-x_c)^2\right]\qquad (0<x<D_0)$$

Maximum pumping rate:

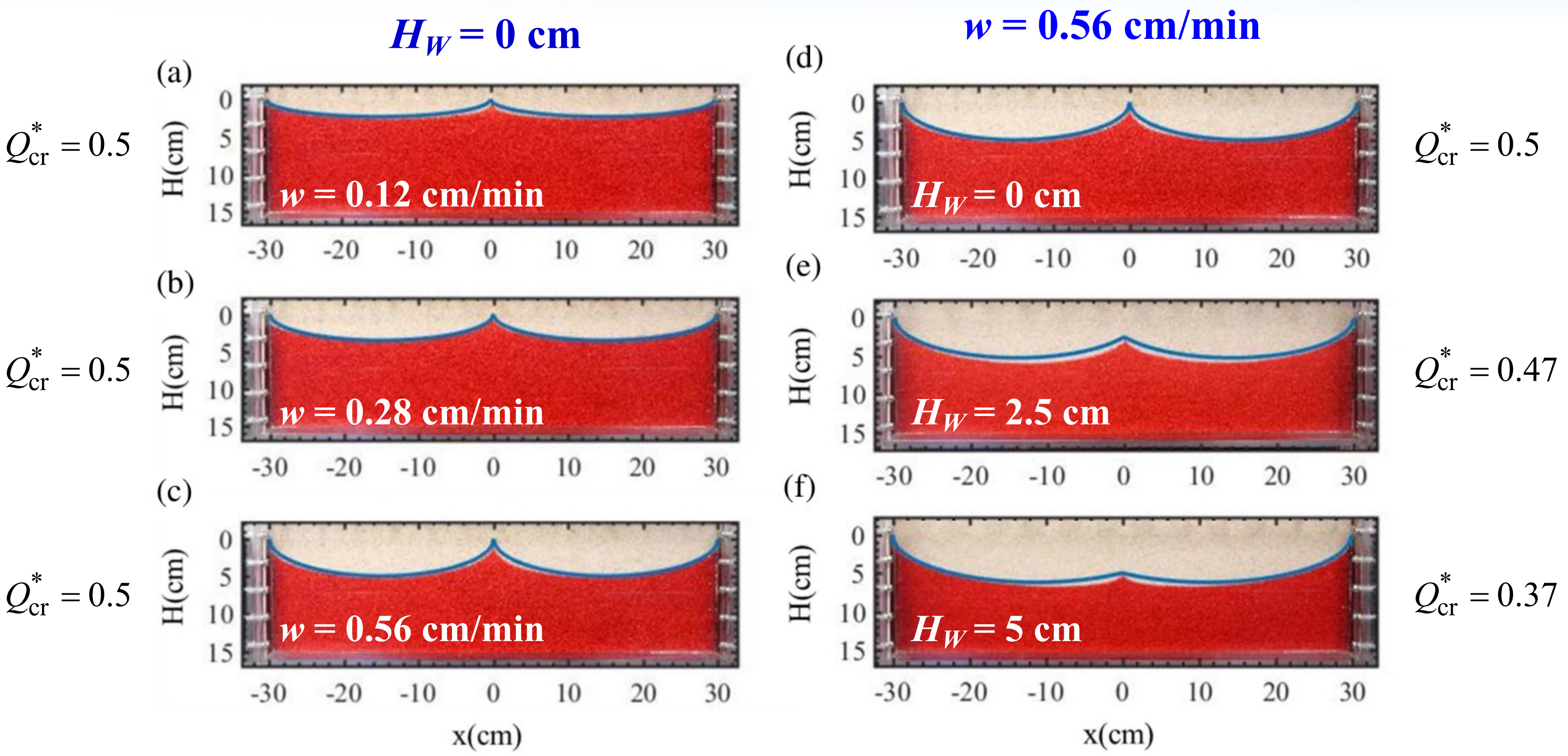
$$q_{cr}=wD\frac{\frac{K_b}{K_0}\left(1-\frac{D_b}{D}\right)^2+\left(2-\frac{D_b}{D}\right)\frac{D_b}{D}-\frac{1+\varepsilon}{\varepsilon^2}\left(\frac{H_W}{D}\right)^2\frac{K_b}{K_0}\frac{K_0}{w}}{\frac{K_b}{K_0}\left(1-\frac{D_b}{D}\right)+\frac{D_b}{D}}$$

*Yan et al*. 2022, WRR

# Laboratory Experiment



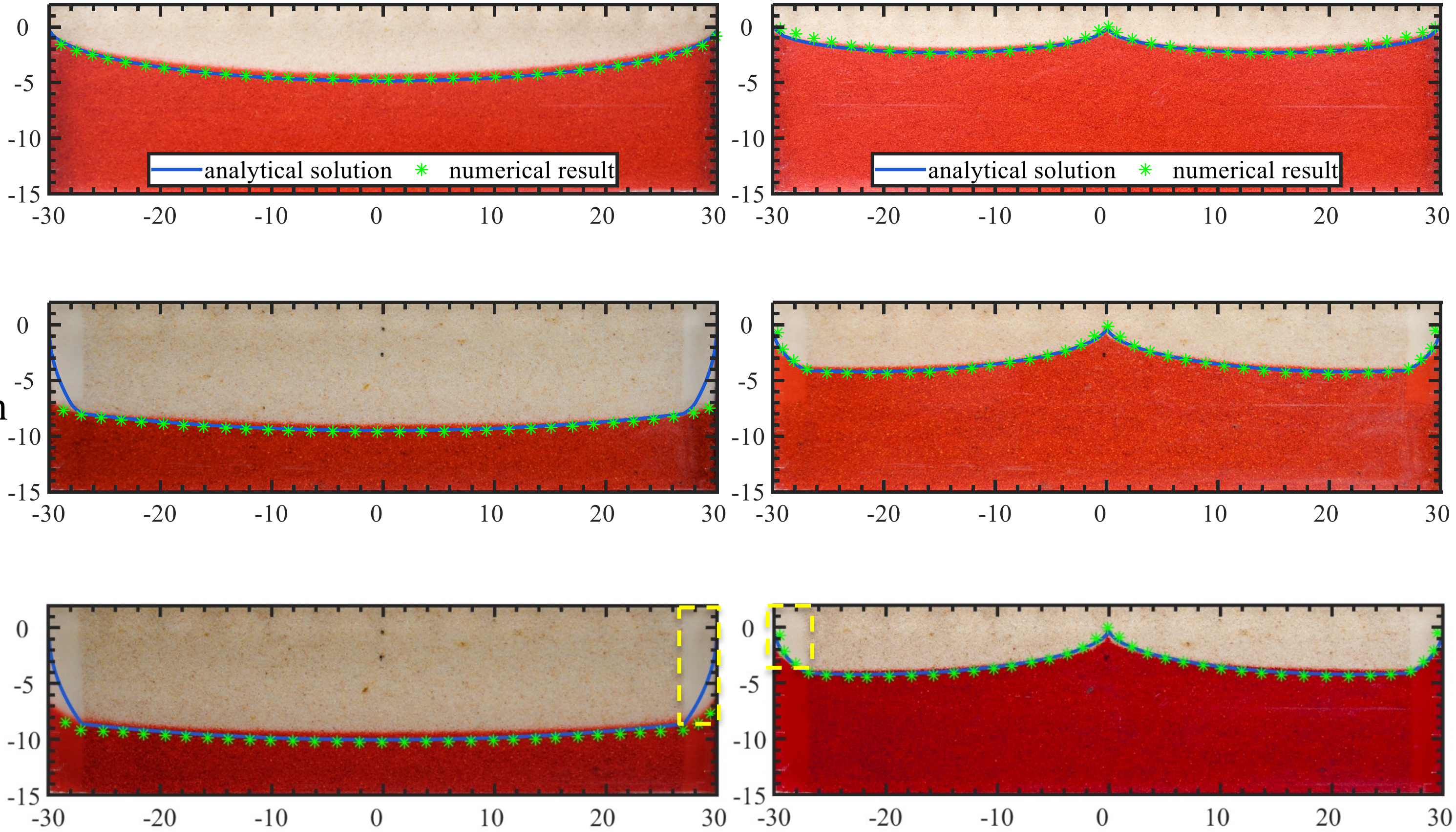
# Maximum Pumping Rate Without Barrier



Maximum pumping rate = 50% of the total recharge, which is achieved when the well located at the center of the island and the same elevation as the sea level.

# Maximum Pumping Rate With Barrier

**Without Pumping** **With Pumping**



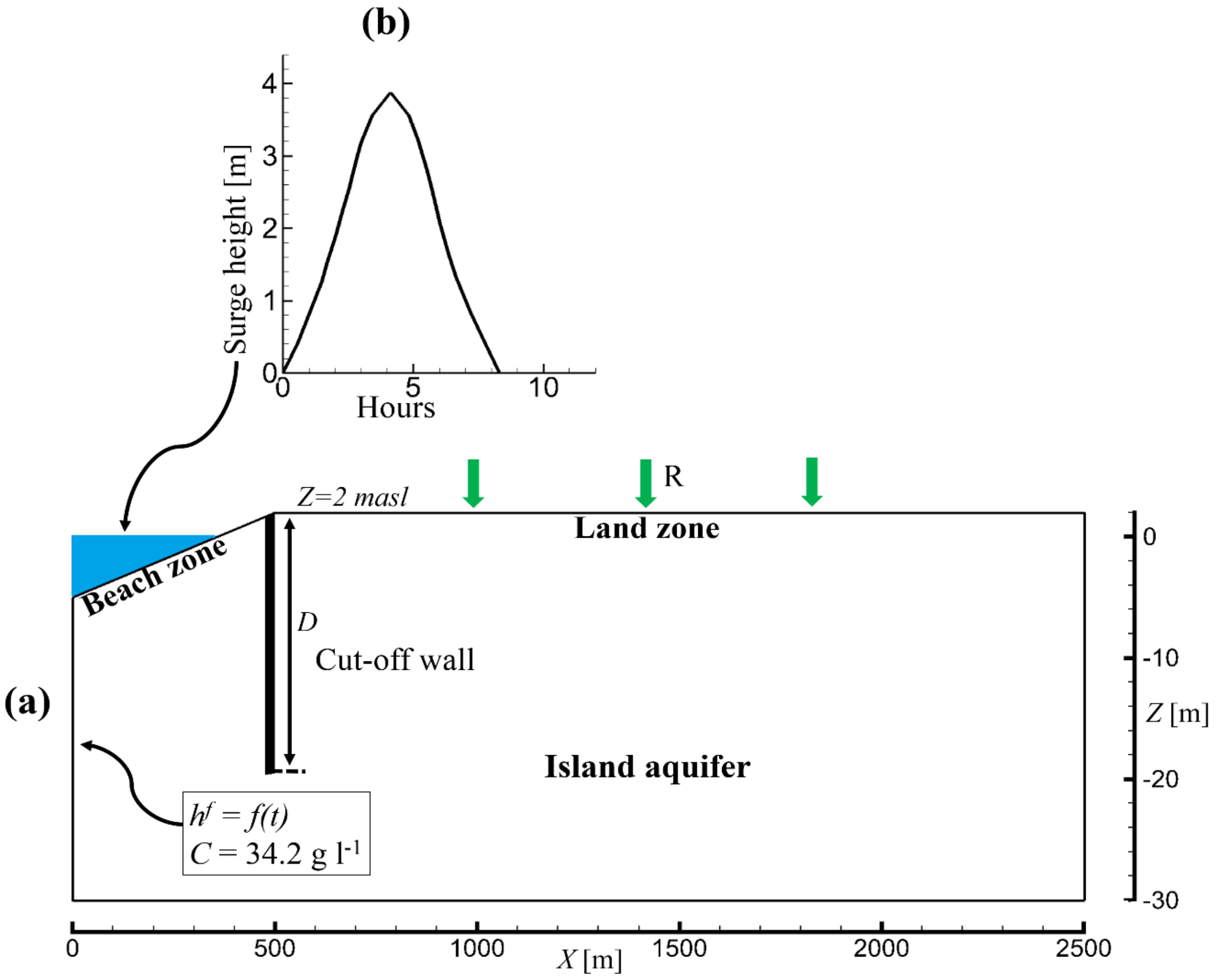
$H_b = 0$ cm $Q^*_{\mathrm{cr}} = 0.5$

$H_b = 7.5$ cm $Q^*_{\mathrm{cr}} = 0.74$

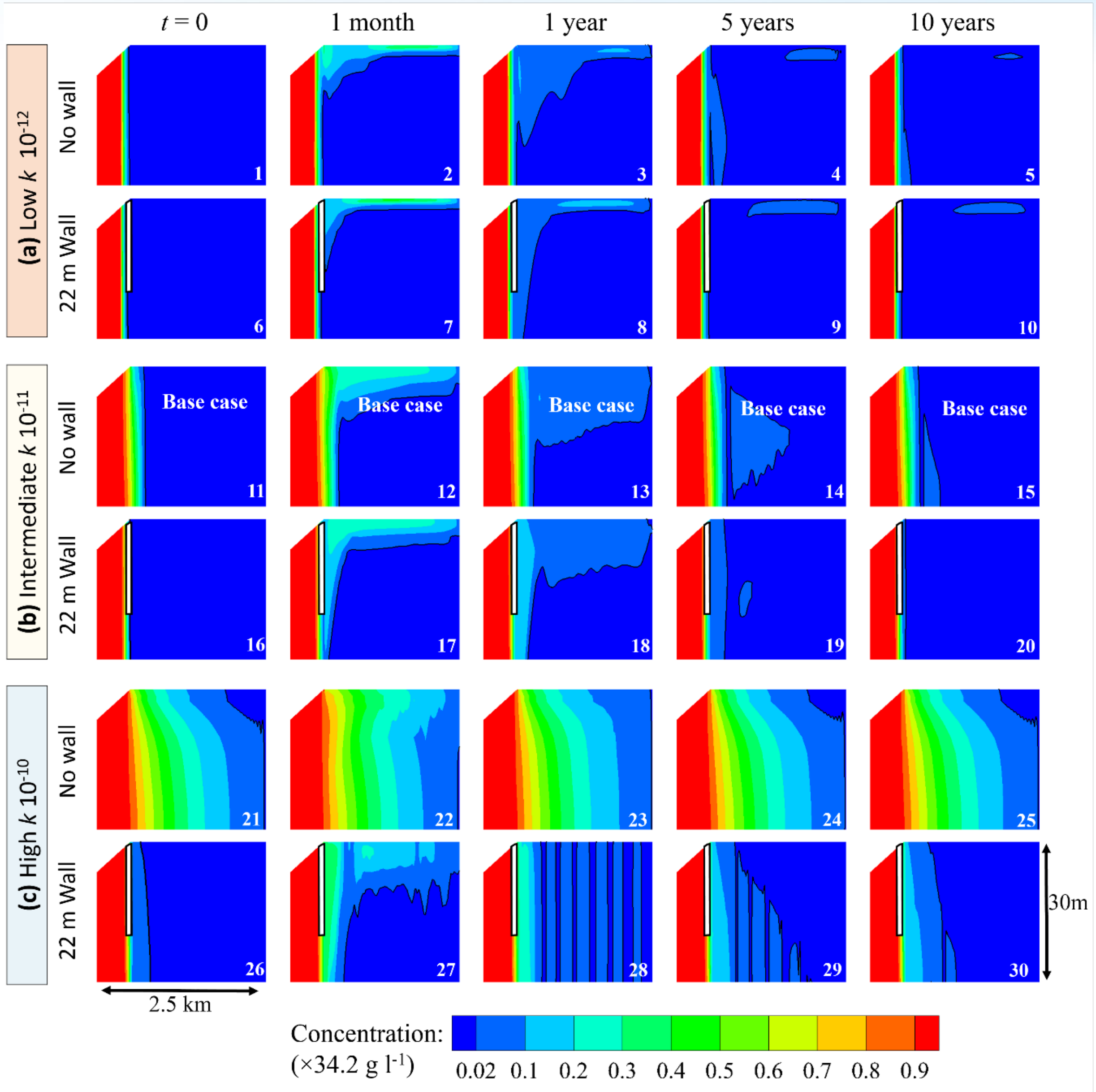
$H_b = 15$ cm $Q^*_{\mathrm{cr}} = 0.74$

# Q3: Will the subsurface barrier enhance or alleviate the effect of storm surge on island groundwater quality?

# Conceptual Model

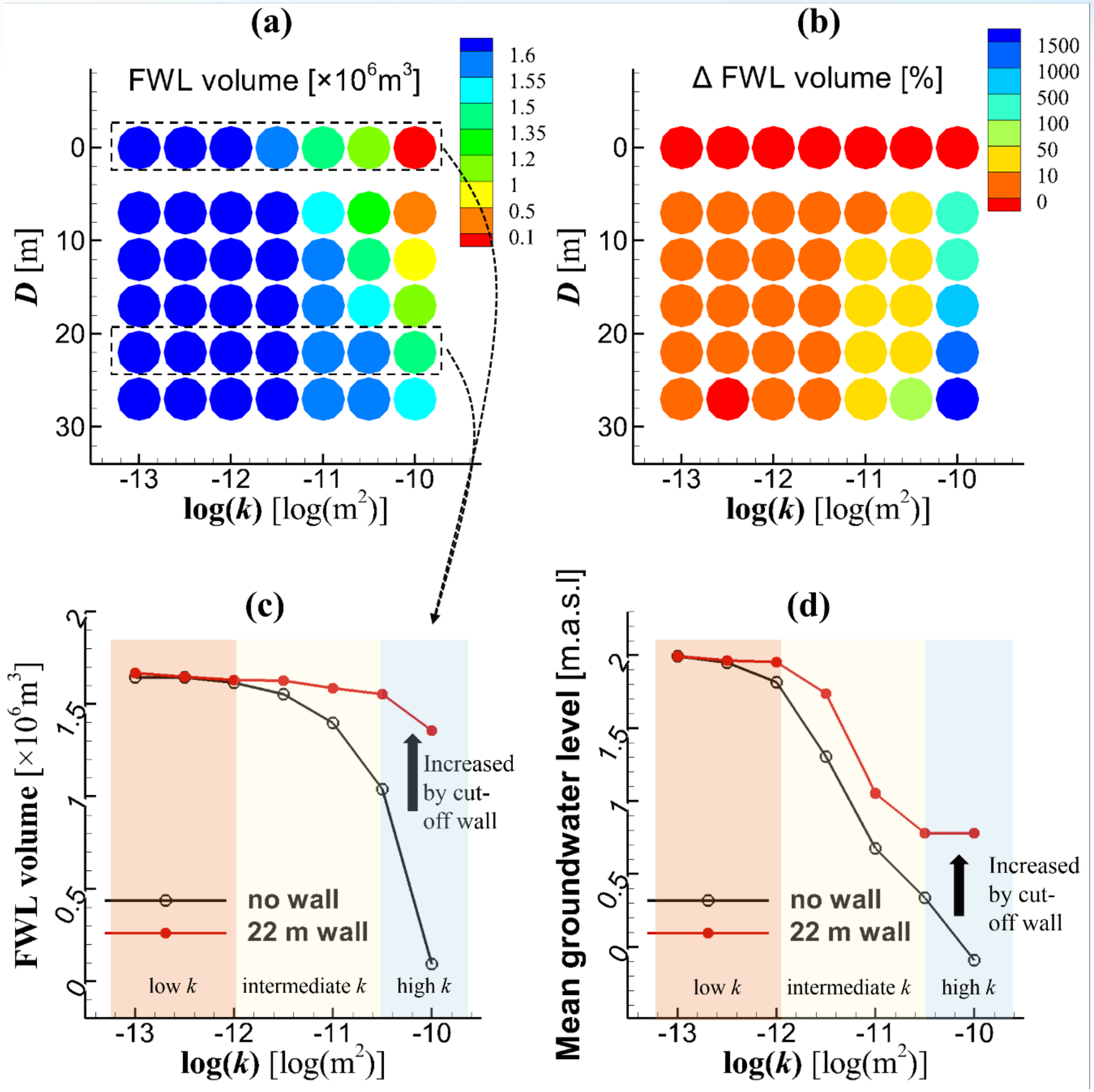


# Storm Surge Effect



*Yang et al*. 2022, JH

# Quantitative Analysis



# Conclusions

- We developed analytical solutions of the freshwater lens for annulus segment island, elliptical island, and islands with spatial and temporal variation in recharge, respectively;
- For the first time, a strategy using a low-permeability barrier is proposed to enhance fresh groundwater storage and extraction in small oceanic islands, and validated through analytical, numerical and experimental results;
- A critical barrier depth is found, indicating that a partially penetrating barrier rather than a fully penetrating barrier could be used to reduce the construction cost without reducing the performance;
- The subsurface barrier alleviates the effect of storm surge on island groundwater quality.

**Thank you for your attention!**