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MATHEMATICAL MODEL OF THE OIL SLICK SPREADING UNDER THE SOLID ICE COVER

Armen Girghidov[1]

ABSTRACT

The mathematical model of oil slick spreading under the solid smooth ice cover under the gravitation and water current is considered. The set of differential equations are analyzed by four numerical methods for Axi-symmetrical and Uni-directional cases. The calculation results are compared to field and laboratory experimental data.

INTRODUCTION

Oil spills under the ice cover occur due to winter navigation and oil drilling in cold climates. Knowledge of oil spills behavior is important for contingency planning in many situations. Simulation of oil spreading under the ice cover is vital to predict the environmental impact after accidents.

Physical modeling of the oil spreading under ice in laboratories does not give a possibility to simulate various field conditions and thus to get a useful information for protection and recuperation measures.

From this point of view mathematical modeling is more productive but models described in available literature of oil interaction with ice give rather limited possibilities. For example some well-known equations (Yapa, 1990) are valid for only axi-symmetrical conditions of oil spreading; so they cannot describe the influence to oil spreading under the shear force from water flow. Rajaratnam's model (1985) includes shear stresses from under ice water flow but he considered only the static problem.

Objective of the present paper is to propose the general model of oil slick movement (advection and spreading) under gravity and shear force from under ice water flow, considering oil spill as a compact body of viscous liquid.

[1] Saint-Petersburg State Polytechnical University, Saint-Petersburg, Russia

MODEL DESCRIPTION

Let the origin of Cartesian coordinates axes (x_1, x_2, x_3) be situated at the oil slick top and x_3 be directed downward (Figure 1.).

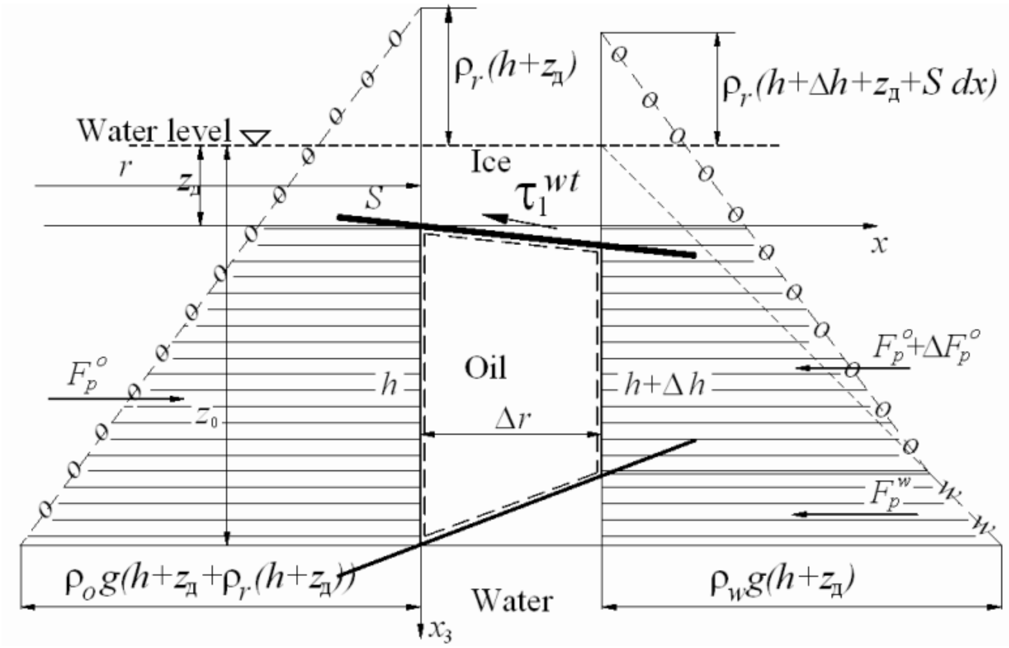

Fig. 1. Control volume

The main assumptions are:

- the motion of oil is laminar;
- pressure distribution is hydrostatic;
- vertical velocity is neglected;
- horizontal components of oil velocity are approximated as:

$$u_i = a_i x_3^2 + b_i x_3 + c_i\,, \tag{1}$$

where $a_i(x_1, x_2, t)$, $b_i(x_1, x_2, t)$, $c_i(x_1, x_2, t)$ – unknown functions; $i = 1,2$.

There are two different ways of the ice-oil interaction.

1. Smooth ice bottom surface. In this case water film forms between oil and ice. Oil slick slips along this water film affected by the under ice water flow. Under this condition $c_i \neq 0$.
2. Ice bottom surface is coarse. In this case the oil velocity at the ice bottom surface equals to zero relative to the ice surface. Thus $c_i = 0$.

Let's consider on first case closer. To evaluate the mean of the water film thickness the water film model is proposed as an addition to oil spreading model. For this model it is assumed:

- ice bottom is horizontal and smooth;

- oil slick has a compact body and its top surface is horizontal and parallel to the ice bottom. The distance between them is $\Delta = \Delta(t)$;
- oil slick moves only in vertical direction under hydrostatic force;
- movement in water film is laminar. Thus, inertial force may be neglected.

Considering water film movement only in horizontal direction x_1 the velocity within water film at any moment t may be presented as in Poiseuille's law (figure):

$$u_1 = \frac{1}{2\eta_w}\frac{\partial p}{\partial x_1}\left(x_3^2 - \Delta x_3\right),$$

where η_w – water dynamic viscosity; p – pressure; Δ – water film thickness.
The water film average speed may be defined as:

$$\mathrm{v}_1(x_1,t) = \frac{1}{\Delta(t)}\int_0^{\Delta(t)} u_1 dx_3 = -\frac{\Delta^2}{12\eta_w}\frac{\partial p}{\partial x_1}.$$

Assuming a linear distribution of the speed from the oil slick center to the borders (axisymmetrical problem) is obtained:

$$\mathrm{v}_1(x_1,t) = \frac{2x_1}{L}\mathrm{v}_{1_0}(t),$$

where L – oil slick length; $\mathrm{v}_{1_0}(t)$ – the border speed of the water film.
Pressing force may be defined as following:

$$N(t) = \frac{2\eta_w \mathrm{v}_{1_0} L^2}{\Delta^2}.$$

From the above the water film thickness is obtained as following:

$$\Delta(t) = \sqrt{\frac{\Delta(0)\eta_w L^3}{2tN\Delta(0) + \eta_w L^3}}.$$

Calculation shows that water film thickness equal to 1 mm becomes to 0.1 mm in 10^4 seconds (about 3 hours).
Taking into account that the only external mass force is gravity, which contributes zero to horizontal force balance, the mass conservation and linear momentum conservation laws may be presented as:

$$\left.\begin{aligned}
&\frac{\partial h}{\partial t}+\sum_{i=1}^{2}\frac{\partial}{\partial x_i}(\mathrm{v}_i\cdot h)=0;\\
&\frac{\partial \mathrm{v}_i h}{\partial t}+\frac{1}{5}\sum_{j=1}^{2}\frac{\partial}{\partial x_i}\left(a_i a_j h^5\right)-\frac{1}{4}\sum_{j=1}^{2}\frac{\partial}{\partial x_i}\left[\left(a_i b_j+a_j b_i\right)h^4\right]-\frac{1}{3}\sum_{j=1}^{2}\frac{\partial}{\partial x_i}\left(b_i b_j h^3\right)+\frac{2}{3}\frac{\partial a_i c_i}{\partial x_i}h^3+=\\
&+\sum_{j=1}^{2}\frac{\partial}{\partial x_2}a_j c_i h^3-\frac{1}{2}\frac{\partial b_i c_i}{\partial x_i}h^2-\frac{1}{2}\sum_{j=1}^{2}\frac{\partial b_j c_i}{\partial x_k}h^2+\sum_{j=1}^{2}\frac{\partial}{\partial x_j}\left(c_i c_j h\right)=\\
&=\frac{\tau_i^{wt}}{\rho_o}+\frac{\tau_i^{wb}}{\rho_o}-g\left[\Delta\cdot\frac{\partial h}{\partial x_i}\rho_r+h\frac{\partial h}{\partial x_i}\left(2\rho_r-1\right)\right]+\nu_o\frac{\partial}{\partial x_k}\left[\frac{1}{3}\sum_{j=1}^{2}\frac{\partial a_j h^3}{\partial x_m}-\right.\\
&\left.-\frac{1}{2}\sum_{j=1}^{2}\frac{\partial b_j h^2}{\partial x_m}+\sum_{j=1}^{2}\frac{\partial c_j h}{\partial x_m}\right],
\end{aligned}\right\}$$

(2)

where h – oil slick thickness; v_i – oil speed ($\mathrm{v}_i=\frac{1}{h}\int_{-h}^{0}u_i dx_3=\frac{1}{h}\int_{-h}^{0}(a_i x_3^2+b_i x_3+c_i)dx_3=a_i\frac{h^2}{3}+b_i\frac{h}{2}=a_i\frac{h^2}{3}+b_i\frac{h}{2}+c_i$); t – time; $\rho_r=\frac{\rho_w-\rho_o}{\rho_o}$ – relative density, where ρ_w – water density; ρ_o – oil density; Δ – water film thickness; ν_o – kinematic oil viscosity; τ_i^{wt} and τ_i^{wb} – shear stresses at the top and bottom of oil slick; $a_i=\frac{b_i}{h}-\frac{\tau_i^{wb}}{\eta_o\cdot h}$; $c_i=\frac{\tau_i^{wt}}{\eta_w}\Delta$

$k=i+(-1)^{i+1}; m=j+(-1)^{j+1}; \ i=1,2$.

Neglecting convective components set of equations is reduced to:

$$\left.\begin{aligned}
&\frac{\partial h}{\partial t}+\sum_{i=1}^{2}\frac{\partial}{\partial x_i}(\mathrm{v}_i\cdot h)=0;\\
&h\frac{\partial \mathrm{v}_i}{\partial t}+g\left[\Delta\cdot\frac{\partial h}{\partial x_i}\rho_r+h\frac{\partial h}{\partial x_i}\left(2\rho_r-1\right)\right]=\frac{\tau_i^{wt}}{\rho_o}+\frac{\tau_i^{wb}}{\rho_o}
\end{aligned}\right\}. \tag{3}$$

BOUNDARY CONDITIONS

Frontal boundary conditions for equations (3) is got from equilibrium of the forces affected to the boundary cell (Fig. 2.):

$$\pm F_{hst}\pm F_{cur}^{vb}\pm F_{cur}^{hb}\pm F_{ice}=0, \tag{4}$$

where F_{hst} – hydrostatic force affected to the boundary cell ($F_{hst} = (\rho_w - \rho_o) g \dfrac{h^2}{2}$); F_{cur}^{vb} – shear force from under ice water flow affected to oil slick vertical border ($F_{cur}^{vb} = \tau_i^{wb} K h$, where K – special coefficient); F_{cur}^{hb} – shear force from under ice water flow affected to oil slick horizontal borders ($F_{cur}^{hb} = \eta_w c_i K$); F_{ice} – shear force from ice bottom surface ($F_{ice} = \eta_o b_i K h$, where η_o – oil dynamic viscosity).

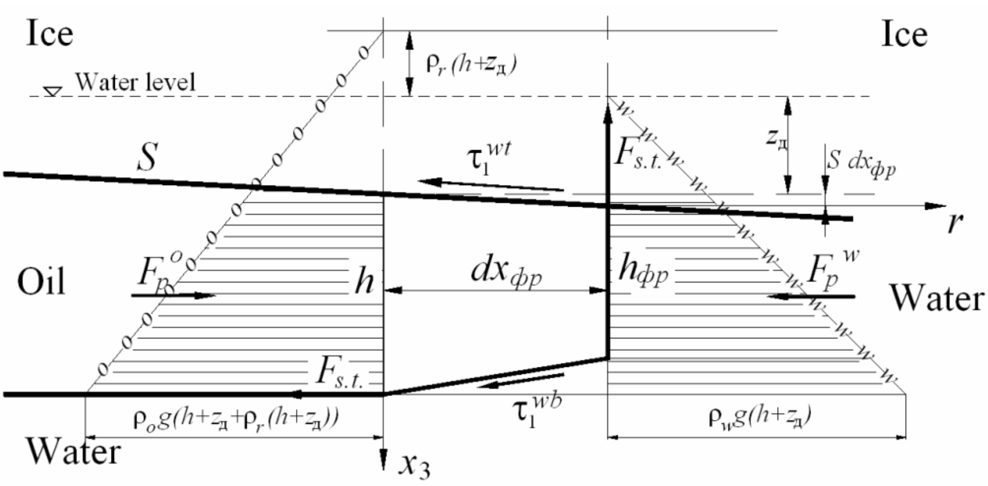

Fig. 2. Boundary conditions in axi-symmerical case

Factor K is response for three-dimensional conditions of flow at the front of the oil slick, and its value should be defined by physical experiments. Substituting forces expressions into equation (4) it can be rewritten as:

$$\pm \rho_r g \frac{h}{2} \pm \nu_o b_i K \pm \nu_w c_i K \pm \frac{1}{\rho_o} \tau_i^{wb} K = 0; \qquad (5)$$

NUMERICAL SOLUTION

The coefficient K was preliminary taken from Yapa’s experimental data for axi-symmetrical spreading. Figure 3 shows comparison between Yapa’s experimental data, calculation by using Yapa’s expression and calculation results by using set of equations (3) with boundary conditions (5).

To verify the K coefficient meaning the calculation results were compared with Alkhimenko’s field experimental data (Alkhimenko et al., 1997) which were successful enough (Table).

	Oil volume in experiment, l	Oil slick head speed at the spreading initial stage, m/s
Alkhimenko et al. experiment (Alkhimenko et al., 1997)	200	$5.5 \cdot 10^{-3}$
Calculation by using set of equations (3) with boundary conditions (5) at oil slick head	200	Decreases from $6.5 \cdot 10^{-3}$ to $5 \cdot 10^{-3}$ during first 10 seconds

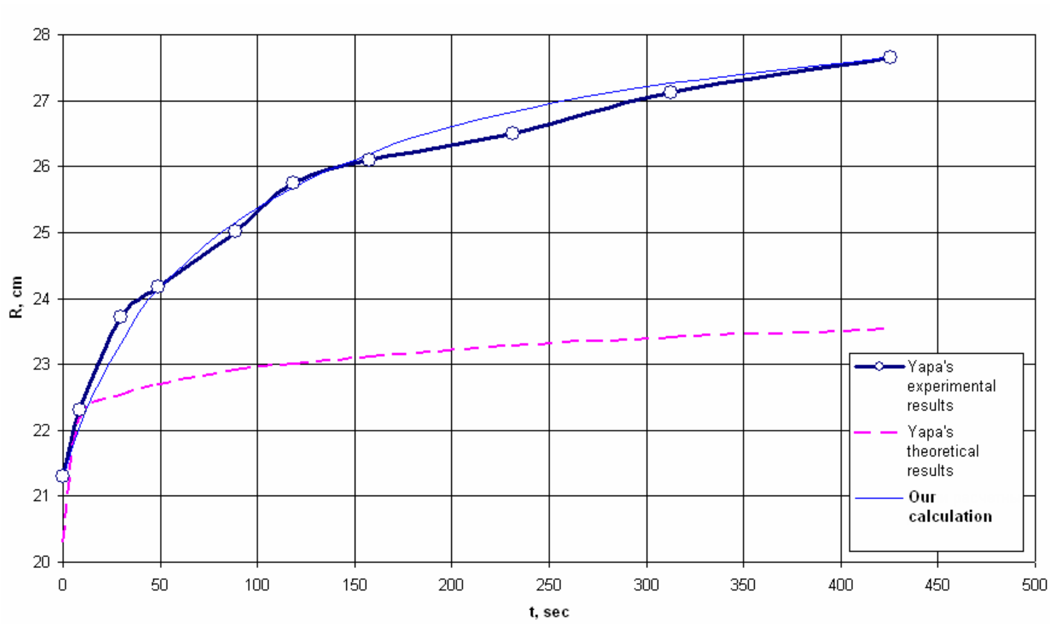

Fig. 3. Compares to Yapa's experimental data (Coefficient $K = 35$)

CONCLUSIONS

The equations presented in this paper can be used to predict the spreading rate during an actual spill or evaluate the potential hazards of spill. Thus, the model presented is more universal than all the usable models in the present time. The model is not snapped to the oil type and water properties. As an addition the different ice bottom contour can be included as initial condition. Two-dimensional model is developed and programmed. It allows calculating the oil spreading and moving in any direction under the ice cover.

REFERENCES

1. Alhimenko, A., et al. Modelling oil pollution under ice cover. *Proceedings of the Seventh International Offshore and Polar Engineering Conference*, Honolulu, USA, Vol. II (1997), pp. 594-601.

2. Berry, B.A., Rajaratnam, N. Oil slicks in ice covered rivers. *Journal of Hydraulic Engineering*, Vol. 111, No. 3, (1985), pp. 369-379.

3. Yapa, P.D., Chowdhury, T. Spreading of oil spilled under ice. *Journal of Hydraulic Engineering*, Vol. 116, No. 12 (1990), pp. 1468-1483.