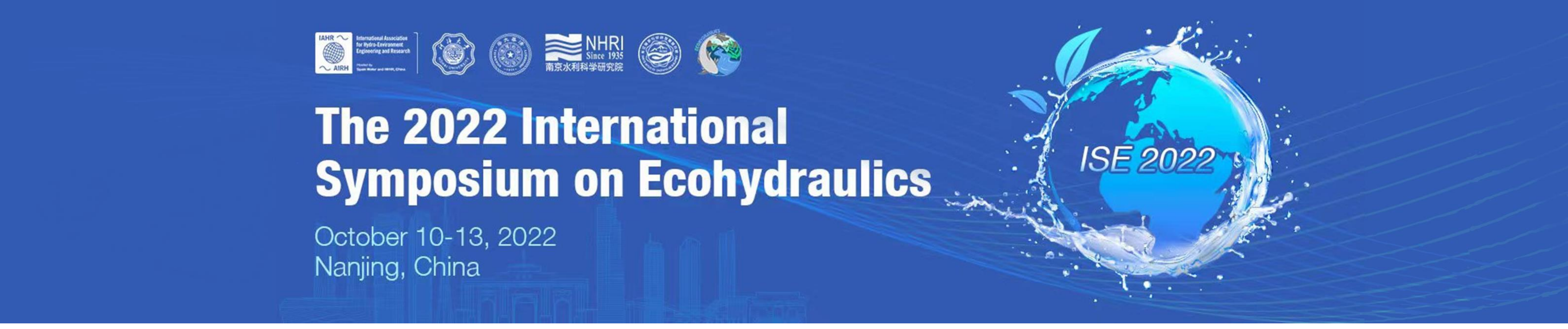

Gyrotactic trapping of micro-swimmers in simple shear flows: A study directly from the fundamental Smoluchowski equation

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Objectives

Thin phytoplankton layers with vertically compressed structures found in stratified lakes and coastal water bodies have been a hot-spot in marine science and fluid mechanics. Although extensive efforts have been made to explore gyrotactic trapping as a possible mechanism, obvious inconsistencies remain there between the generalized Taylor dispersion method and individual-based model. In this work, a study directly from the fundamental Smoluchowski equation is carried out on the gyrotactic trapping mechanism in representative simple shear flows.

Methods

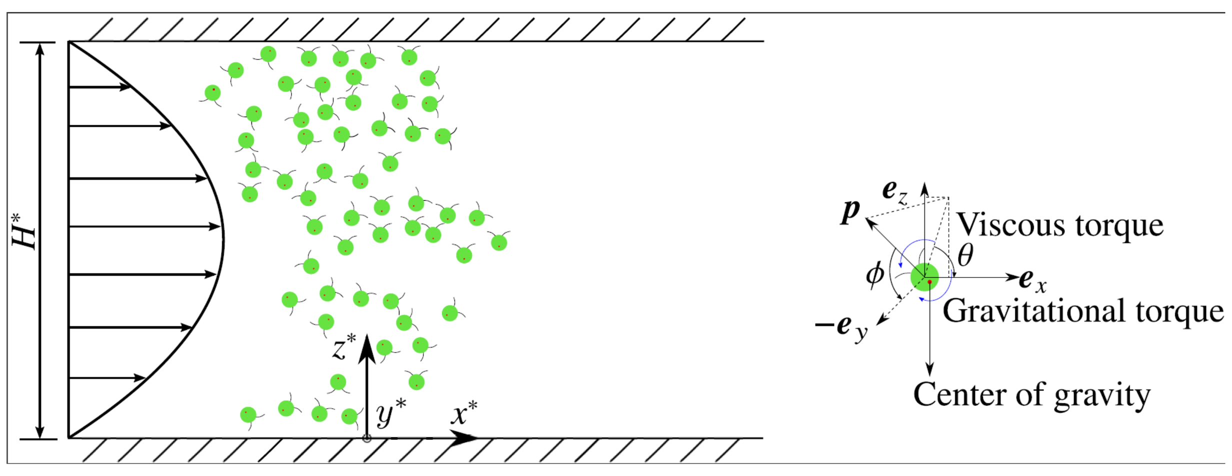

Figure 1. Sketch of the problem.

The problem setting

- *Governing equation (Smoluchowski equation)*

$$\frac{\partial P}{\partial t} = -[Pe_s \sin\phi\cos\theta + Pe_f U(z)]\frac{\partial P}{\partial x} + D_t\frac{\partial^2 P}{\partial x^2} + \mathcal{L}_u P,$$

- *Reflective boundary conditions*

$$-2Pe_d P|_{\theta=\theta_0} - D_t\frac{\partial P}{\partial z}\bigg|_{\theta=\theta_0} - D_t\frac{\partial P}{\partial z}\bigg|_{\theta=2\pi-\theta_0} = 0 \quad \text{on } z=0,1, \quad \forall\,\theta_0\in[0,2\pi).$$

Solution procedure

- Transformation of the p.d.f.

$$P(x,z,\phi,\theta,t) = f_r(z)G^r(x,z,\phi,\theta,t),$$
$$f_r(z) = \exp\left(-\frac{Pe_d}{D_t}z\right).$$

- Classical B.C. for the transformed p.d.f.

$$\left.\begin{aligned} G^r|_\theta &= G^r|_{2\pi-\theta}, \\ \frac{\partial G^r}{\partial z}|_\theta &= -\frac{\partial G^r}{\partial z}|_{2\pi-\theta}, \end{aligned}\right\} \quad \text{on} \quad z=0,1, \quad \forall\,\theta\in[0,2\pi)$$

- Basis functions

$$\left.\begin{aligned} &N_n\sqrt{\frac{2l+1}{4\pi}}\cos(n\pi z)\mathrm{P}_l(\cos\phi), \\ &N_n\sqrt{\frac{2l+1}{2\pi}\frac{(l-m)!}{(l+m)!}}\cos(n\pi z)\cos(m\theta)\mathrm{P}_l^m(\cos\phi), \\ &N_n\sqrt{\frac{2l+1}{2\pi}\frac{(l-m)!}{(l+m)!}}\sin(n\pi z)\sin(m\theta)\mathrm{P}_l^m(\cos\phi), \end{aligned}\right\}$$

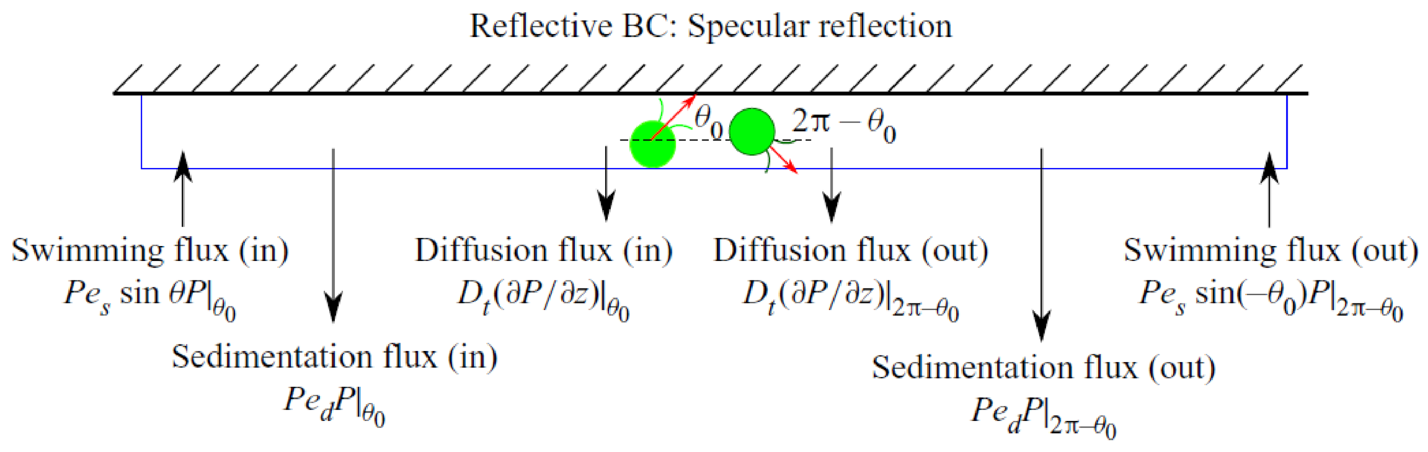

Figure 2 Illustration of the reflective boundary conditions.

- Deriving the eigenfunction and dual basis-functions by a Galerkin method
- Biorthogonal expansion of moments

Results

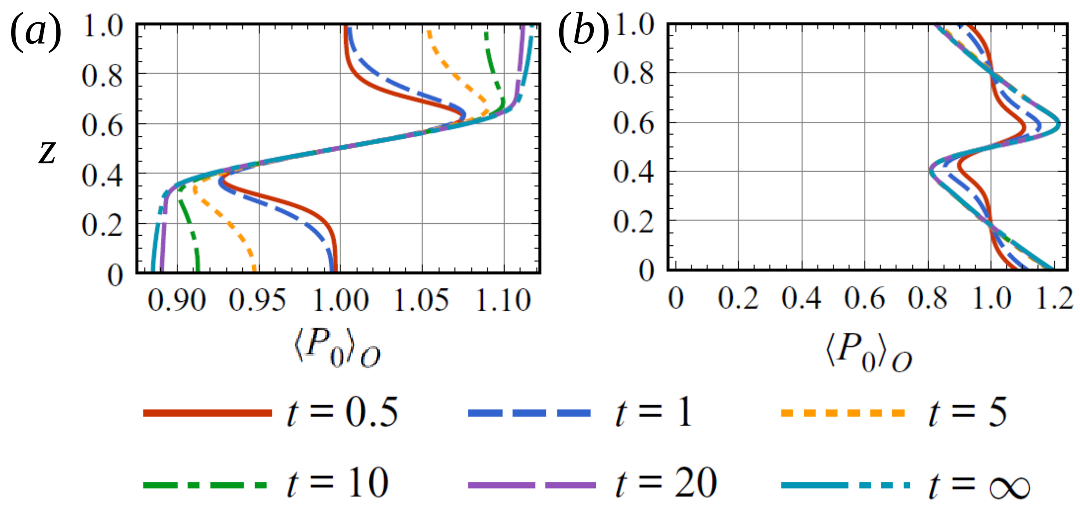

Figure 3. Transient vertical distribution of a vertically uniform line-source release of (*a*) nonsettling (*b*) settling gyrotactic micro-swimmers in the plane Poiseuille flow.

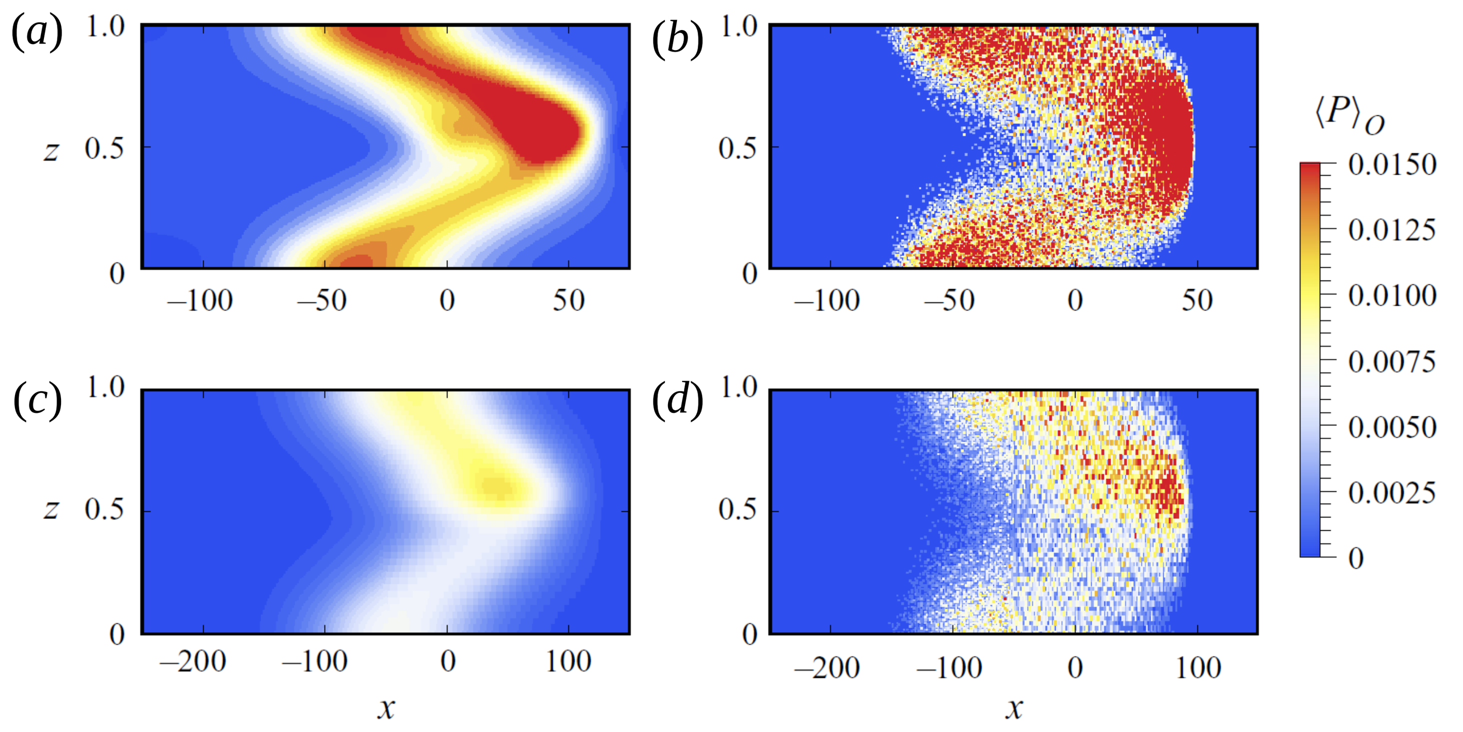

Figure 4. Transient two-dimensional concentration distributions of a vertically uniform line-source release of non-settling gyrotactic micro-swimmers in a strong plane Poiseuille flow. Left: Edgeworth expansion; right: individual-based model (IBM). Top: $t = 5$; bottom: $t = 10$.

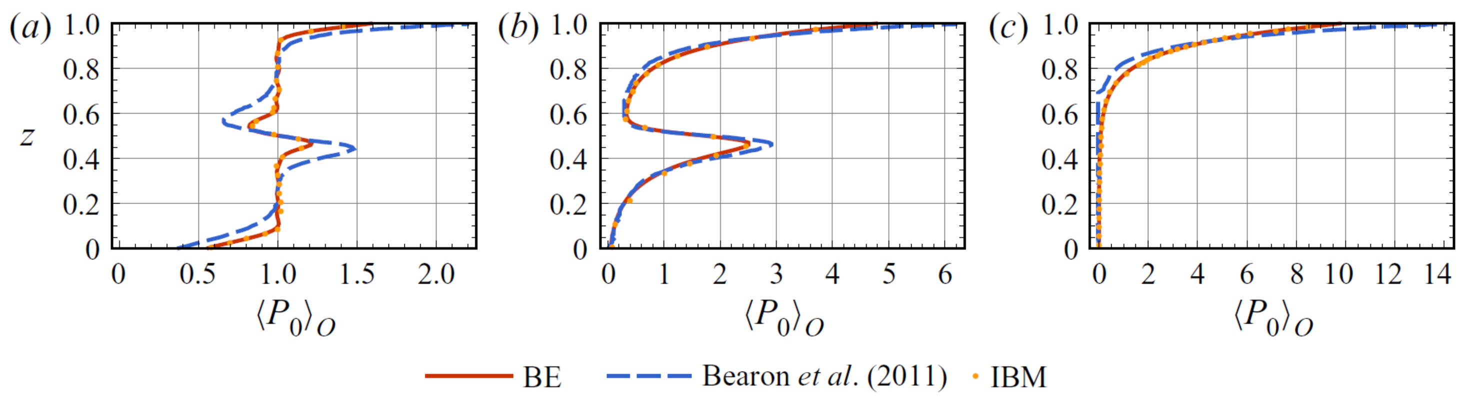

Figure 5. Comparison of vertical distributions of a vertically uniform line-source release of non-settling gyrotactic micro-swimmers between the current model (BE), Bearon et al. (2011) and IBM.

Conclusions

We have given a more complete picture of gyrotactic trapping. Our results are in good agreement with the IBM simulations due to no approximation made to the fundamental Smoluchowski equation. We found that a steady thin layer can be realized in the solution by adding a small settling speed to counteract the gravitactic focusing at the upper surface.